Experiments with fractals

Julia_Sets applet

To run the *Julia_Sets* applet, start MATLAB and type `Julia_Sets` at the prompt. The program iterates the quadratic map $f(z) = z^2 + c$, where z and c are complex numbers. The (filled-in) Julia set is the set of initial conditions z_0 such that successive iterates $z_1 = f(z_0)$, $z_2 = f(z_1)$, …, $z_{n+1} = f(z_n)$, … remain bounded. Points in the Julia set are shown in dark blue. Points z_0 outside the Julia set are color-coded according to how fast successive iterates of z_0 go to infinity.

The sliders at the bottom allow you to change the real and imaginary parts of the parameter c; the `Iterations` slider allows you to set the number of successive iterations used in deciding whether points belong to the Julia set. Of course, the Julia set displayed in the applet is a numerical approximation of the true Julia set. The `Resolution` panel controls the number of points actually plotted in the picture. You have to keep in mind that a higher resolution will lead to a nicer picture, but the code will be slower.

Understanding Julia sets

To understand how Julia sets are defined, pick a value of c and calculate by hand the iterates of simple points, such as the origin (0,0), or (1,0) or (0,1). To do so, you will have to convert the quadratic map defined above as a transformation on the real and imaginary parts of c. Show your work below.

- Express the quadratic map $f(z)$ as a map $g(x,y)$ on the real and imaginary parts of $z = x + \mathrm{i}\, y$.
- Choose a value of the complex number c.
- Write down the successive iterates of z_0, where z_0 has the following values.
    - $z_0 = (0,0)$
    - $z_0 = (1,0)$
    - $z_0 = (0,1)$
- Decide whether each of the points z_0 above is in the Julia set of c.

Review of complex numbers

Read the “just-in-time” information on complex numbers and do a few of the associated exercises. The point of this review is to make sure you are comfortable with the algebra of complex numbers. Show your work below.

Exploring Julia sets

- Describe how the Julia set changes when the value of c is varied. Choose a value of *c* which gives you a nice Julia set and paste below a picture of this Julia set.
- Describe how the number of iterations affects a given Julia set.
- Zoom in on the Julia set to explore its self-similar properties. Paste below pictures of the same Julia set at different magnifications and comment on your observations. Do not forget to include the values of the parameters (*c*, number of iterations, resolution, number of magnifications) used to plot the pictures.

Fractals in nature

- Give examples of self-similarity in nature.
- Add pictures of self-similar objects observed in nature (include references). Explain why they can be considered as self-similar, and give an estimate of the number of levels of magnification over which the object remains self-similar.