Math 250B (Tuesday Problems Session - 2/24/09)

Problem 1. Consider the sequence defined by

$$a_{n+1} = a_n + \frac{1}{4}, \qquad n \geq 1, \qquad a_1 = 1.$$

1. Write the first five terms of the sequence.
2. Find an expression for a_n in terms of n only, and check your result against the terms you calculated in part 1.

Problem 2. [From *Introduction to Mathematics for Life Scientists*, by E. Batschelet, Springer, 3rd Edition, page 166.] A female moth (*Tinea pellionella*) lays nearly 150 eggs. In one year, there may live up to five generations. Each larva eats about 20 mg of wool. Assume that 2/3 of the eggs die and that 50% of the remaining moths are females. Estimate the amount of wool that may be destroyed by the descendants of one female within a year. (The first female belongs to the first generation.)

Problem 3. [From the *Calculus* book, #6 page 469.] Find an expression for the general term of the series below. Make sure the starting value of the index is correct.

$$px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \dots$$

Problem 4. [From the *Calculus* book, # 18 page 469.] Use the ratio test to find the radius of convergence of the following power series.

$$\frac{x}{3} + \frac{2x^2}{5} + \frac{3x^3}{7} + \frac{4x^4}{9} + \frac{5x^5}{11} + \dots$$

Problem 5. [From the *Calculus* book, # 2, 4, 6, 29, 39 page 474.] Decide whether the following statements are true or false. In each case, justify your answer by giving an explanation (if the statement is true) or a counter-example (if the statement is false).

1. If the terms s_n of a convergent sequence are all positive, then $\lim_{n\to\infty} s_n$ is positive.
2. If the sequence s_n of positive terms is unbounded, then the sequence has an infinite number of terms greater than one million.
3. If a series converges, then the sequence of partial sums of the series also converges.
4. If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} (-1)^n |a_n|$ converges.
5. If a power series converges at one endpoint of its interval of convergence, then it converges at the other.