Second Order ODEs - Review Problems

April 13, 2009

1. Find a particular solution to

$$x^5 y'' + 3xy' + 7y = 14.$$

2. Consider the following equation

$$\frac{d^2x}{dt^2} + 25\ x = 16\cos(3t).$$

   (a) Find the general solution of this equation.

   (b) Show that the particular solution satisfying $x(0) = 0$ and $\dot{x}(0) = 0$ is $x_p(t) = \cos(3t) - \cos(5t)$.

   (c) Using the identity $\cos(3t) - \cos(5t) = 2\sin(4t)\sin(t)$, sketch the graph of the particular solution found in (b) for $0 \leq t \leq 2\pi$.

3. Solve the differential equation

$$y'' + 6y' + 9y = \frac{1}{x}e^{-3x}.$$

4. Answer the following questions about the solution $x(t)$ to the damped equation

$$m\ddot{x} + k\dot{x} + hx = 0,$$

   where m, k and h are positive constants.

   (a) If $m = 2$, how should h and k be related so that the non-zero solutions are oscillatory?

   (b) If $h = k = 1$, how should the mass m be chosen so that all non-zero solutions will oscillate?

   (c) If $m = h = 1$, how should k be chosen so that $x(t)$ is oscillatory?

5. Solve the differential equation

$$y'' - 3y' + 2y = -\frac{e^{2x}}{e^x + 1}.$$

Hint: $1/(e^t + 1)$ can be integrated by noticing that it is equal to $e^{-t}/(1 + e^{-t})$.

6. (a) Find a particular solution to the equation

$$\frac{d^2x}{dt^2} + 22\ x = \cos(\gamma t),$$

where $\gamma > 0$. Note that your answer should depend on γ. For what value of γ is there resonance?

(b) Find a particular solution $x_p(t)$ to

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 22\ x = \cos(\gamma t).$$

What is the value of x_p at $t = \pi/(2\gamma)$? Sketch the behavior of this value as a function of γ. What happens when $\gamma = \sqrt{22}$?

7. Solve the differential equation

$$y'' + 2y' + y = \frac{2}{x^2}e^{-x}.$$