First Order Systems

This lab is concerned with second order linear equations with constant coefficients and their associated first order systems. To fix the ideas, consider the differential equation

$$a\,y'' + b\,y' + c\,y = 0, \tag{1}$$

where $a \neq 0,\ b,\ c, \in \mathbb{R}$. This equation may be re-written (WHY?) as the following first-order system

$$\begin{cases} y' = v \\ v' = -\dfrac{b}{a}v - \dfrac{c}{a}y \end{cases}. \tag{2}$$

Our goal is to relate the behavior of solutions $y(t)$ of Equation (1) to properties of the trajectories of system (2) in the $(y,\ v)$ plane. For each of the examples below,

1. Find the general solution to the given differential equation.
2. Choose a few different initial conditions and plot the corresponding trajectories in the $(y,\ v)$ plane (recall that $v = y'$).
3. Make a general statement about the nature of the trajectories in the $(y,\ v)$ plane for the particular type of differential equation that is being discussed.

Example 1: $y'' - 5\,y' + 6\,y = 0$

Example 2: $y'' + 9\,y = 0$

Example 3: $y'' - 6\,y' + 10\,y = 0$

Example 4: $y'' + y' - 6\,y = 0$