Calculus and Differential Equations I

MATH 250 A

Methods of integration I

Two methods of integration

- Integration by substitution is a method based on the chain rule:

$$\int f'(g(x))\, g'(x)\, dx = \int \frac{d}{dx} f(g(x))\, dx = f(g(x)) + C.$$

- Examples: Find $\int (2x+1)\, e^{x^2}\, e^x\, dx$ and $\int_1^4 \frac{\cos(\sqrt{x})}{\sqrt{x}}\, dx$.

- Integration by parts is a method based on the product rule:

$$\int u(x)\, v'(x)\, dx = u(x)\, v(x) - \int u'(x)\, v(x)\, dx.$$

- Examples: Find $\int x\, \sinh(x)\, dx$ and $\int \ln(t)\, dt$.

Which method to use?

- When to make a substitution: roughly speaking, if the integrand involves a function of a complicated expression, it is a good idea to define this complicated expression as a new variable.

- When to use integration by parts: this method will "work" if the integral of $u'(x)\, v(x)$ is somewhat simpler than that of $u(x)\, v'(x)$. Otherwise, there is no reason to use integration by parts. Note that sometimes one has to use integration by parts more than once.

- How would you evaluate the integral $\int x^3 \sin(x)\, dx$.
  1. Integration by parts
  2. Integration by substitution
  3. None of the above

Which method to use? (continued)

- How would you evaluate the integral $\int \frac{x^2}{1+x^2}\, dx$.
  1. Integration by parts
  2. Integration by substitution
  3. None of the above

- How would you evaluate the integral $\int \frac{x^2}{1+x^3}\, dx$.
  1. Integration by parts
  2. Integration by substitution
  3. None of the above

- How would you evaluate the integral $\int x^4 \exp(x^3)\, dx$.
  1. Integration by parts
  2. Integration by substitution
  3. None of the above

Some of your questions

- If one has to make a substitution to evaluate a definite integral, what should one do about the limits of integration?
  1. Leave them unchanged
  2. Recalculate them in terms of the new variable
  3. First find an antiderivative and then use the fundamental theorem of Calculus to evaluate the definite integral
  4. Either 1 or 3
  5. Either 2 or 3
  6. None of the above
- Can you find an example of an integral that can be evaluated both by substitution and by integration by parts?
  1. Yes
  2. No

Some of your questions (continued)

- If we cannot evaluate an integral, does it mean that the integrand does not have an antiderivative?
  1. Yes
  2. No
- If one tries to use integration by parts in a situation where the integral can only be evaluated by substitution, is the method of integration by parts going to give a wrong answer?
  1. Yes
  2. No
- If one has to use integration by parts repeatedly to evaluate a definite integral, is it better to evaluate the brackets as one goes along, or to find an antiderivative first and then evaluate the definite integral?