MATH 223 TEST #4

December 1st, 1998

1. Calculate $\int_S \vec{F} \cdot d\vec{A}$, where $\vec{F} = 5x\vec{i} + 3\vec{j}$ and

   (a) (8 points) S is the square of side 2 perpendicular to the y-axis, centered on the y-axis, and oriented in the negative y direction.

   (b) (12 points) S is the square of side 2 perpendicular to the x-axis, centered on the x-axis, and oriented in the positive x direction.

2. (20 points) Calculate the flux of the vector field $\vec{F} = 5\vec{r} + \dfrac{\vec{r}}{||\vec{r}||^3}$ through the sphere of radius 3 centered at the origin.

3. Find $\int_C \vec{F} \cdot d\vec{r}$, where C is the triangle whose vertices are $(0,0,0)$, $(1,0,0)$ and $(0,1,0)$, which is oriented counterclockwise, and $\vec{F}$ is

   (a) (20 points) $\vec{F} = -2xy\vec{i} + x^2\vec{j}$

   (b) (20 points) $\vec{F} = 2x\vec{i} + (y+1)\vec{j} + z\vec{k}$

4. Consider the curve C parametrized by $\vec{r}(t) = e^t\ \vec{i} - e^{-t}\ \vec{j} + t\vec{k}$, for $0 \leq t \leq 1$.

   (a) (3 points) Find the velocity vector $\vec{v}(t)$ of a particle whose position is given by $\vec{r}(t)$.

   (b) (3 points) Find the acceleration vector $\vec{a}(t)$ of the particle.

   (c) (14 points) Find $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (x^2 - y^2)\ \vec{i} + 2xyz\ \vec{j} - 2yz\vec{k}$.