MATH 223 - 05 Test # 3

29 October 1998

1 Part I [30 points]

1. Set up triple integrals giving the volume of the solids between the plane $z = 1$ and the graph of $z = 4 - \sqrt{x^2 + y^2}$, and above the following regions in the xy-plane:

   (a) the square $-1 \leq x \leq 1$, $-1 \leq y \leq 1$

   (b) the disk $x^2 + y^2 \leq 1$.

   You can use the coordinates of your choice. Do not evaluate the integrals.

2. **Extra credit:** Evaluate the integral found in (b) above.

2 Part II [20 points]

Evaluate $\int_0^2 \int_y^2 \cos(x^2)\, dx\, dy$.

3 Part III [25 points]

1. Give a parametrization of the following curves:

   (a) The circle of radius 3, centered at the origin, traced out counter clockwise and located in the xy-plane.

   (b) The line connecting the points $P_1 : (1, 2, 3)$ and $P_2 : (-1, 0, 4)$, oriented from P_1 to P_2.

2. Does the line $x = 2t$, $y = 2t$, $z = t$ cross the ellipsoid $x^2 + 2y^2 + 3z^2 = 15$? If so, at what points ?

4 Part IV [25 points]

Find the mass of a sphere of radius 1 and of density $D(\rho) = \rho^2$, where ρ measures the distance between a point in the sphere and the center of the sphere.