MATH 223 - 05 Test # 2

8 October 1998

1 Part I [35 points]

Consider the surface $z = f(x, y)$, given as the graph of a function f. Let

$$\vec{u} = \frac{1}{5}(3\vec{i} + 4\vec{j}) \qquad \text{and} \qquad \vec{v} = \frac{1}{5}(4\vec{i} - 3\vec{j}).$$

Suppose that

$$f_{\vec{u}}(1,3) = 5; \qquad f_{\vec{v}}(1,3) = 0; \qquad f(1,3) = 5.$$

1. Show that the gradient of f at $(1,3)$ is $\nabla f(1,3) = 3\vec{i} + 4\vec{j}$.
2. Find the equation of the tangent plane to the surface $z = f(x, y)$ at $(1,3,5)$.
3. Find the differential of f at $(1,3)$.
4. Find the directional derivative of f at $(1,3)$ in the direction of the vector $\vec{w} = \vec{i} + 5\vec{j}$.

2 Part II [30 points]

Consider the ellipsoid $(x-1)^2 + 4(y-2)^2 + (z-3)^2 = 17$.

1. Find the equation of the tangent plane to the ellipsoid at the point $(3,3,6)$.
2. Find all of the points on the ellipsoid where a vector normal to the ellipsoid is perpendicular to the plane $z = 5$.

3 Part III [35 points]

1. Find the four second-order partial derivatives of

$$f(x, y) = x^2 e^y + \ln\left(\frac{y}{x}\right).$$

2. If $z = f(x, y)$ and $x = s + t$, $y = s - t$, show that

$$\left(\frac{\partial z}{\partial t}\right)\left(\frac{\partial z}{\partial s}\right) = (f_x)^2 - (f_y)^2.$$

3. If $z = 3xy + 7x^3 + e^y$, and $x = e^u$, $y = ue^w$, find $\dfrac{\partial z}{\partial w}(1, 1)$ using the chain rule.