MATH 223 - 05 Test # 1

23 September 1998

1 Part I [25 points]

1. The planes of equation $x + y + z = 4$ and $3x - 3y + z = 0$ are parallel. True or false ? Justify your answer.
2. The contour diagram of $f(x, y) = \sqrt{x^2 + y^2}$ consists of evenly-spaced concentric circles. True or false ? Justify your answer.
3. The table below corresponds to a linear function. True or false ? Justify your answer.

x \ y	10	15	20
-5	33	28	23
0	23	18	13
5	10	5	0

4. The point P with coordinates $(1, 2, -5)$ is on the plane of equation $x + y - 3z = 5$. True or false ? Justify your answer.
5. The equation $y^2 - x^2 - z^2 = 0$ describes two cones, centered on the y-axis, with vertex at the origin. True or false ? Justify your answer.

2 Part II [20 points]

Given $\vec{u} = 4\vec{i} - 3\vec{j} + \vec{k}$ and $\vec{v} = 2\vec{i} + \alpha\vec{j} - \vec{k}$, find $\vec{u} \cdot \vec{v}$ and $\vec{u} \times \vec{v}$ (show all your work). Can you find α such that $\vec{u}$ and $\vec{v}$ are

1. perpendicular ?
2. parallel ?

3 Part III [35 points]

1. For each of the functions below, plot (as functions of y) the cross-sections of $f(x, y)$ at $x = -1$, $x = 0$ and $x = 1$. Put labels on your plots.

   (a) $f(x, y) = 3x - 7y + 5$

   (b) $f(x, y) = x^2 - y^2 + 1$

   In each case, describe (in words) the surface of equation $z = f(x, y)$.

2. Draw a contour diagram of the function of two variables obtained by taking a cross-section of $g(x, y, z) = x^2 + y^2 - z$ with $z = 4$. Show at least 4 level curves with labels.

3. Draw a contour diagram of $h(x, y) = -y^2$. Show at least 4 level curves with labels.

4 Part IV [20 points]

Given the points $P = (1, 2, 3)$, $Q = (1, 5, 7)$ and $R = (-1, -1, -1)$, find:

1. The equation of the sphere of radius 5, centered at P. Is Q on the sphere ?

2. The unit vector in the direction of $\vec{PR}$.

3. The equation of the plane perpendicular to $\vec{QR}$ and going through P.