# Drill problems on derivatives and antiderivatives

## 1 Derivatives

Find the derivative of each of the following functions (wherever it is defined):

1. $f(t) = \dfrac{t^2 + t^3 - 1}{t^4}$

   **Answer:** $f'(t) = \dfrac{-2}{t^3} - \dfrac{1}{t^2} + \dfrac{4}{t^5}$

2. $y = \dfrac{1}{3\sqrt{x}} + \dfrac{1}{4}$

   **Answer:** $\dfrac{dy}{dx} = -\dfrac{1}{6x\sqrt{x}}$

3. $f(t) = 2t^3 - 4t^2 + 3t - 1$. Also find $f''(t)$.

   **Answer:** $f'(t) = 6t^2 - 8t + 3,\ f''(t) = 12t - 8$

4. $y = \sqrt{x} - \left(\dfrac{1}{2}\right)^x$

   **Answer:** $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}} + \ln(2)\left(\dfrac{1}{2}\right)^x$

5. $f(z) = \ln(3)z^2 + \ln(4)e^z$

   **Answer:** $f'(z) = 2\ln(3)z + \ln(4)e^z$

6. $y = x^{\pi^2} + (\pi^2)^x$

   **Answer:** $\dfrac{dy}{dx} = \pi^2 x^{\pi^2 - 1} + [\ln(\pi^2)](\pi^2)^x$

7. $f(\theta) = 4^{\sqrt{\theta}}$

   **Answer:** $f'(\theta) = \ln(4)\dfrac{1}{2\sqrt{\theta}}4^{\sqrt{\theta}}$

8. $f(x) = \left(x^2 - \sqrt{x}\right) 3^x$

   **Answer:** $f'(x) = \left(2x - \dfrac{1}{2\sqrt{x}}\right) 3^x + \left(x^2 - \sqrt{x}\right) \ln(3) 3^x$

9. $f(z) = \dfrac{3z^2}{5z^2 + 7z}$

   **Answer:** $f'(z) = \dfrac{21}{(5z+7)^2}$

10. $f(w) = (5w^2 + 3)e^{w^2}$

    **Answer:** $f'(w) = 10we^{w^2} + (5w^2+3)2we^{w^2} = (5w^2+8)2we^{w^2}$

11. $f(y) = e^{e^{y^2}}$

    **Answer:** $f'(y) = 2ye^{y^2}e^{e^{y^2}}$

12. $f(z) = \dfrac{\sqrt{z}}{(e^z+1)^2}$

    **Answer:** $f'(z) = \dfrac{1}{2\sqrt{z}} \dfrac{1}{(e^z+1)^2} - \dfrac{2\sqrt{z}e^z}{(e^z+1)^3}$

13. $w(t) = (t^2 + 3t)(1 - e^{-2t})$

    **Answer:** $w'(t) = 2t + 3 + e^{-2t}(2t^2 + 4t - 3)$

14. $f(x) = \sqrt{1 - \cos(x)}$

    **Answer:** $f'(x) = \dfrac{\sin(x)}{2\sqrt{1-\cos(x)}}$

15. $f(y) = e^{\sin(y)}$

    **Answer:** $f'(y) = \cos(y)e^{\sin(y)}$

16. $z = \sqrt{\sin(t)}$

    **Answer:** $\dfrac{dz}{dt} = \dfrac{\cos(t)}{2\sqrt{\sin(t)}}$

17. $f(\theta) = \theta^2 \sin(\theta) + 2\theta\cos(\theta) - 2\sin(\theta)$

    **Answer:** $f'(\theta) = \theta^2 \cos(\theta)$

18. $z = \tan(e^{-3\theta})$

    **Answer:** $\dfrac{dz}{d\theta} = -3\sec^2(e^{-3\theta})e^{-3\theta}$

19. $y = e^{\theta}\sin(2\theta)$

    **Answer:** $\dfrac{dy}{d\theta} = e^{\theta}\sin(2\theta) + 2e^{\theta}\cos(2\theta)$

20. $f(y) = \arcsin(y^2)$

    **Answer:** $f'(y) = \dfrac{2y}{\sqrt{1-y^4}}$

21. $f(\theta) = \ln(\cos(\theta))$

    **Answer:** $f'(\theta) = -\tan(\theta)$

22. $f(t) = \ln(\ln(t)) + \ln(\ln(2))$

    **Answer:** $f'(t) = \dfrac{1}{t\ln(t)}$

23. $g(t) = \arctan(3t-4)$

    **Answer:** $g'(t) = \dfrac{3}{1+(3t-4)^2}$

24. $f(z) = \dfrac{1}{\ln(z)}$

    **Answer:** $f'(z) = \dfrac{-1}{z(\ln(z))^2}$

25. $f(t) = 2te^t - \dfrac{1}{\sqrt{t}}$

    **Answer:** $f'(t) = 2e^t + 2te^t + \dfrac{1}{2t\sqrt{t}}$

# 2 antiderivatives

Find the definite and indefinite integrals below:

1. $\displaystyle\int \left(\frac{3}{t} - \frac{2}{t^2}\right)\, dt$

   **Answer:** $3\ln(|t|) + \dfrac{2}{t} + C$

2. $\int \left(3\cos(\psi) + 3\sqrt{\psi}\right)\ d\psi$

   **Answer:** $3\sin(\psi) + 2\psi\sqrt{\psi} + C$

3. $\int \left(\dfrac{x^2 + x + 1}{x}\right)\ dx$

   **Answer:** $\dfrac{1}{2}x^2 + x + \ln(|x|) + C$

4. $\int (3\cos(x) - 7\sin(x))\ dx$

   **Answer:** $3\sin(x) + 7\cos(x) + C$

5. $\int \dfrac{1}{\cos^2(x)}\ dx$

   **Answer:** $\tan(x) + C$

6. $\int e^{\sin(x)}\cos(x)\ dx$

   **Answer:** $e^{sin(x)} + C$

7. $\int_0^{\pi/4} (\sin(t) + \cos(t))\ dt$

   **Answer:** $1$

8. $\int \sin(\theta)\left(\cos(\theta) + 5\right)^7\ d\theta$

   **Answer:** $\dfrac{-1}{8}\left(\cos(\theta) + 5\right)^8 + C$

9. $\int \dfrac{1}{\sqrt{4 - x}}\ dx$

   **Answer:** $-2\sqrt{4 - x} + C$

10. $\int xe^{-x^2}\ dx$

    **Answer:** $-\dfrac{1}{2}e^{-x^2} + C$

11. $\int \dfrac{x\cos(x^2)}{\sqrt{\sin(x^2)}}\ dx$

    **Answer:** $\sqrt{\sin(x^2)} + C$

12. $\displaystyle\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$

**Answer:** $\displaystyle \ln\left(\frac{e^x + e^{-x}}{2}\right) + C$

13. $\displaystyle\int \frac{[\ln(z)]^2}{z}\, dz$

**Answer:** $\displaystyle \frac{1}{3}[\ln(z)]^3 + C$

14. $\displaystyle\int_0^1 \frac{1}{x^2 + 2x + 1}\, dx$

**Answer:** $\displaystyle \frac{1}{2}$

15. $\displaystyle\int_{-2}^0 \frac{2x + 4}{x^2 + 4x + 5}\, dx$

**Answer:** $\ln(5)$

16. $\displaystyle\int t\sin(t)\, dt$

**Answer:** $-t\cos(t) + \sin(t) + C$

17. $\displaystyle\int y\sqrt{y+3}\, dy$

**Answer:** $\displaystyle \frac{2}{3}(y+3)\sqrt{y+3}\left[\frac{3}{5}y - \frac{6}{5}\right] + C$

18. $\displaystyle\int t^2 e^{5t}\, dt$

**Answer:** $\displaystyle \left(t^2 - \frac{2t}{5} + \frac{2}{25}\right)\frac{e^{5t}}{5} + C$

19. $\displaystyle\int_0^1 \arctan(y)\, dy$

**Answer:** $\displaystyle \frac{\pi}{4} - \frac{\ln(2)}{2}$

20. $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\, dx$

**Answer:** $-\sqrt{1-x^2} + C$