MATH 223 - 03 Test # 4

3 December 1997

Part I. [20 points]

The position of a particle as a function of time t is given by

$$x = \cos(3e^t), \qquad y = \sin(3e^t), \qquad z = e^t.$$

1. Find the velocity and acceleration vectors of the particle as functions of t.

2. What is the length of the curve traced by the particle for $0 \leq t \leq 2\pi$?

Part II. [40 points]

Consider the surface parametrized by

$$x = \cos(s-t) \qquad y = \sin(s-t) \qquad z = s+t.$$

1. Find $\int_{C_1} \vec{F} \cdot d\vec{r}$, where

$$\vec{F} = \left(yz\vec{i} + xz\vec{j} + xy\vec{k}\right)\cos(xyz),$$

and C_1 is the parameter curve given by $s = 0$, $0 \leq t \leq 2\pi$.

2. Describe the curve C_2 corresponding to $s = -t$, $0 \leq t \leq \pi$.

3. Find the circulation of $\vec{F}$ around C_2.

4. Find the circulation of $\vec{G} = x\vec{j}$ around C_2.

Part III. [20 points]

Compute the flux of $\vec{F} = x^2\vec{i} + y^2\vec{j} + z\vec{k}$ through the open cylinder of radius 2 and height 3, centered on the z-axis with its base in the xy-plane, and oriented outward.

Part IV. [20 points]

Compute the flux of $\vec{F} = -xz\vec{i} - yz\vec{j} + z^2\vec{k}$ through the surface given by $z = 4 - x^2 - y^2$, $z \geq 0$, oriented upward.