MATH 223 - 06 Practice Test # 3

Part I. [25 points]

A cylindrical tube of radius 2 cm and length 3 cm contains a gas. Since the tube is spinning around its axis, the density of the gas increases with the distance from the axis. The density D at a distance of r cm from the axis is $D(r) = (1 + r)$ g/cm^3.

1. Write a triple integral representing the total mass of the gas in the tube.

2. Evaluate this integral.

Part II. [20 points]

Consider the integral

$$I = \int_{1/2}^{1} \int_{x}^{\sqrt{3}\ x} f(x, y)\, dy\, dx.$$

1. Draw a picture of the region R over which integration is performed.

2. Rewrite I in polar coordinates.

Part III. [30 points]

Compute the following integral

$$I = \int_{0}^{1/\sqrt{2}} \int_{-\sqrt{\frac{1}{2}-x^2}}^{\sqrt{\frac{1}{2}-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} dz\, dy\, dx.$$

Hint: Draw a picture and change coordinates !

Part IV. [25 points]

Consider the curve C drawn in Figure 1. Carefully sketch the behavior of the coordinates $x(t)$ and $y(t)$ of a particle moving counter clockwise on C, starting from the point $(2, 0)$ at $t = 0$. Label by $t_1, t_2, \ldots$ the times where you have to change the parametrization and mark on C the position of the particle at these given times.

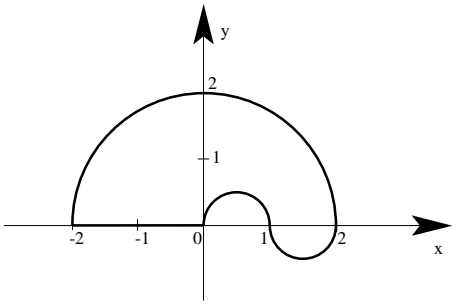

Figure 1: Curve C