MATH 223 - 06 Test # 2

27 February 1997

Part I. [45 points]

1. Find the gradient vectors and differentials of the following functions of two and three variables:

$$(a) \quad f(x,y) = (x+y)e^y$$
$$(b) \quad g(x,y,z) = z(x^2+y^2)$$

Evaluate dg at $(0,2,3)$.

2. Find g_{zx}, g_{zy} and g_{zz}. What can you say about g_{yz} and g_{xz} ?

3. Let $x = u\ v$, $y = \ln(v)$ and $z = f(x,y) = (x+y)e^y$. Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.

Part II. [15 points]

Suppose that when moving away from the point $(0,0,0)$, the function $h(x,y,z)$ increases most rapidly in the direction of the vector $\vec{i}+\vec{j}+\vec{k}$, and that the rate of change of h in this direction is equal to 3. What is the rate of change of h when you move away from $(0,0,0)$ in the direction of the vector $\vec{v} = 3\vec{i}+\vec{j}-\vec{k}$?

Part III. [40 points]

Consider the surface of equation $z(x^2+y^2) = 5$.

1. What does this surface look like ?

2. Find the equation of the tangent plane at $(1,2,1)$, using two different methods: first by considering the surface as a level surface of some function of 3 variables $F(x,y,z)$; second by viewing the surface as the graph of a function of 2 variables $z = f(x,y)$. Explain what you are doing.

3. Find all the points on the surface where a vector perpendicular to the surface is in the (y,z) plane.

4. Is there any point on the surface where a vector perpendicular to the surface is parallel to the z axis ? Why ?