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- We will use abstract symbols for propositions (p,q,r,...)

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- In logic, this distinction is called inclusive ($\vee$) or exclusive ($\oplus$)

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- Negation of "I am going to the store, and I am studying math."?

- Device to show all possible values of proposition
- Examples: disjunction, conjunction, negation, exclusive or

Let
p = this class is hard
q = the homework is easy
r = the exams are long

A) "This class is hard but the homework is easy."
B) "The exams are short, despite the homework being difficult."
C) $p \vee \neg q$

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- Truth tables for $\neg p \wedge \neg q$, $(p \vee q) \vee \neg r$

1. Suppose you want to automatically open window blinds from 8am to 5pm, except weekends and holidays. Let

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Answer: $(p \wedge q) \wedge \neg r \wedge \neg s$.

2. "To rent an apartment, you need a credit score above 500, and either 2 references or a bank account with 10000 dollars.
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Answer: $\neg(p \vee q)$ or $\neg p \wedge \neg q$.

Is there a systematic way of doing this?

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- Conclusion need not be related to hypothesis:
  "If it is January, then elephants are large"
  (Is the conditional true? Are p and q true?)

- Given $p \to q$, there are three other related conditionals:
  Converse: $q \to p$
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Chapter 1.3 Related conditional statements

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- $p =$ "A month has 28 days", $q =$ "A month is February". Is $p \leftrightarrow q$ always true? Can q be adjusted to make it so?
- Hidden biconditionals: "If the restaurant does not have pasta, I will eat the pizza."

Examples: truth tables for compound expressions

(A) $(p \to q) \to (q \to p)$

(B) $(p \vee q) \leftrightarrow (q \to p)$

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Translating to symbols from English:

P: a person can vote

Q: a person is at least 18 yrs

R: a person is a resident of the state

(A) "A person can vote if they are a state resident and 18 yrs. old."

(B) "If someone can vote, they are 18 yrs. old"

(C) "If someone can vote, they either are not 18 or not a resident"

(D) "If a person can vote, then if they are 18 they must also be a state resident."

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  For example: "This class is not fun or interesting" is same as "This class is not fun and this class is not interesting." (show by writing symbolic version)

Idempotent laws:	$p \vee p \equiv p$	$p \wedge p \equiv p$
Associative laws:	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Commutative laws:	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Distributive laws:	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Identity laws:	$p \vee \mathbf{F} \equiv p$	$p \wedge \mathbf{T} \equiv p$
Domination laws:	$p \wedge \mathbf{F} \equiv \mathbf{F}$	$p \vee \mathbf{T} \equiv \mathbf{T}$
Double negation law:	$\neg\neg p \equiv p$	
Complement laws:	$p \wedge \neg p \equiv \mathbf{F}$ $\neg \mathbf{T} \equiv \mathbf{F}$	$p \vee \neg p \equiv \mathbf{T}$ $\neg \mathbf{F} \equiv \mathbf{T}$
De Morgan's laws:	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Conditional identities:	$p \rightarrow q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

All can be verified by truth tables.

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$\equiv T \wedge (\neg p \vee r) \wedge (T \vee \neg r)$ (commutative, compliment)
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$\equiv p \to r$ (identity, conditional)

(A) Simplify $\neg(p \vee (\neg p \wedge q))$.

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(B) Show $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$.

(C) Why is "If today is sunny, then if tonight is rainy then today is sunny" a tautology?

(A) Simplify $\neg(p \vee (\neg p \wedge q))$. Answer: $\equiv \neg p \wedge \neg q$.

(B) Show $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$.

(C) Why is "If today is sunny, then if tonight is rainy then today is sunny" a tautology? Hint: Consider $p \rightarrow (r \rightarrow p)$.