Math 160 Section 1
Summation Notation

I. Overview

We need an efficient symbolic method to denote the sum of a lot of terms, each of which depends on an integer in a sequence of integers, such as the sum of the first 100 integers. We will use this in calculations of various statistical entities, such as the mean, $\overline{X}$. Summations have very wide use in statistics.

II. Two solutions to this symbolic problem have been devised

A. The Ellipsis

An elipsis is a series of centered dots which indicate missing terms in the sum. Usually the beginning terms are given, followed by the dots, followed by the general form to be added, followed by a few of the terms on the end. For the sum of the first 100 integers, this first form of a large sum looks like:

$$1 + 2 + 3 + \cdots + i + \cdots + 98 + 99 + 100 = 5050$$

This notation is long to write and somewhat cumbersome so mathematicians, ever conscious of ink and writing labor, have devised one more compact.

B Sigma Notation

The name comes from the name for the Greek letter $\Sigma$ which equivalent to a capital S in English (the minuscule is $\sigma$.) The sum is written in Sigma form as

$$\sum_{i=i_0}^{n} a_i$$

where $i_0$ indicates that we are to start the sum at this value (the first term), $n$ indicates where the sum is to stop (the last term), and $a_i$ is the form to be used for each term. More formally, $\Sigma$ is a summation operator, $i$ is the index, $i_0$ is the lower limit, $n$ the upper limit, and $a_i$ is the summand; the whole thing being called a summation or a sum. So the above example is written in Sigma notation as

$$\sum_{i=1}^{100} i = 5050$$

since

$$\begin{aligned} i_0 &= 1 \\ n &= 100 \\ \text{and } a_i &= i \end{aligned}$$

The $i$ is often used for the index, but $j$, $k$, and other letters are used interchangeably. There is no reason to start at 1 or end at 100, or to use such a simple function of the index, thus

$$\sum_{j=5}^{7} i^2 = 5^2 + 6^2 + 7^2 = 110$$

III. The following algebraic rules apply, and you should convince yourself that they make sense.

$$\sum_{j=1}^{n} a_j + b_j = \sum_{j=1}^{n} a_j + \sum_{j=1}^{n} b_j \tag{1}$$

$$\sum_{j=1}^{n} r \cdot a_j = r \sum_{j=1}^{n} a_j \tag{2}$$

$$\sum_{j=1}^{n} a_j = \sum_{j=1}^{m} a_j + \sum_{j=m+1}^{n} a_j \qquad 1 \leq m \leq n \tag{3}$$

$$\sum_{j=1}^{n} a_j = \sum_{j=1+c}^{n+c} a_{j-c} \qquad c > 1 \tag{4}$$

The first is an addition rule, the second is a distributive law. The next (#3) is concerned with splitting a sum, the last with shifting the index.
Our first contact with Sigma notation is early in the text when we encounter numerical descriptors of the center of a distribution:

$$\overline{X} = \frac{1}{n} \sum_{i=1}^{n} x_i.$$

There are lots more!