FINAL EXAM STUDY GUIDE – MATH 115B SPRING 2003

1. Data on the test markets and costs for a good are given below. All monetary amounts are in dollars and all quantities are single units.

Potential National Market: 2,500			
Test Markets			
Market Number	Market Size	Price	Projected Yearly Sales
1	100	$59.95	50
2	300	$89.95	45
3	200	$69.95	80
4	500	$39.95	350
5	400	$79.95	120

Cost Data	
Fixed Cost: $1,000	
Variable Costs	
Quantity	Cost per unit
First 500 units	$5
Next 500 units	$3
Further	$2

(a) Use the data in Test Market 2 to compute the number of units in the national market that would be sold at a price of $89.95.

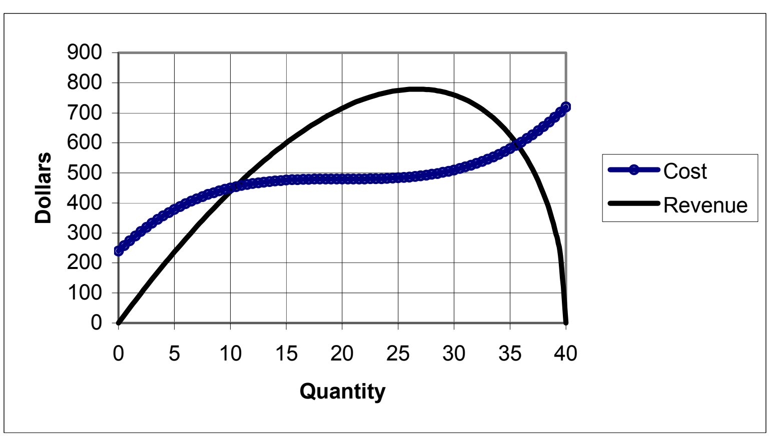
(b) The equation for the polynomial trend line that has been fitted to the data in the five test markets is given by $f(x) = -0.00009x^2 - 0.0303x + 100.32$. Use this equation to estimate the price per unit if 600 units are produced and sold.

(c) How much revenue would be earned if 600 units are produced and sold?

(d) What would be the total cost of producing 600 units?

(e) How much profit would be earned if 600 units are produced and sold?

2. The graphs below represent the cost and revenue for a particular product. Use the graphs to estimate the following:



(a) the number of units that need to be sold so that the profit is zero.

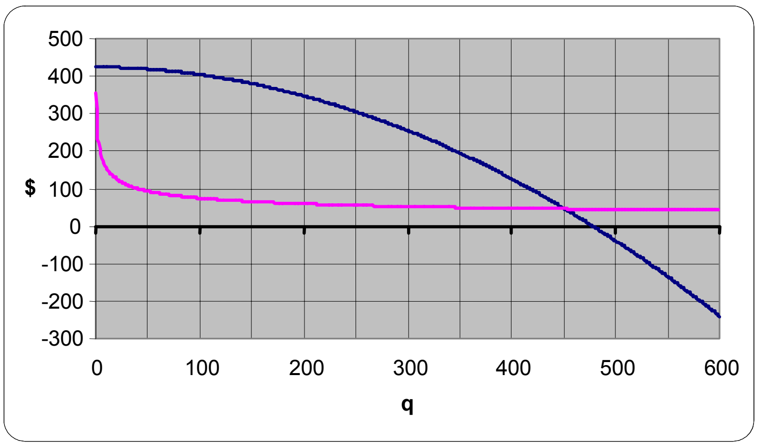
(b) the fixed costs.

(c) the number of units that need to be sold to maximize profit.

(d) the maximum profit.

(e) the revenue where the marginal revenue function is equal to zero.

3. Graphs of the marginal revenue and marginal cost functions for a product are given below.



(a) Is the revenue function increasing or decreasing at $q = 200$?

(b) Is the cost function increasing or decreasing at $q = 200$

(c) Is the revenue function increasing or decreasing at $q = 500$?

(d) Is the cost function increasing or decreasing at $q = 500$?

(e) Is the profit function increasing or decreasing at $q = 250$?

(f) For what value of q is profit maximized?

(g) For what value of q is revenue maximized?

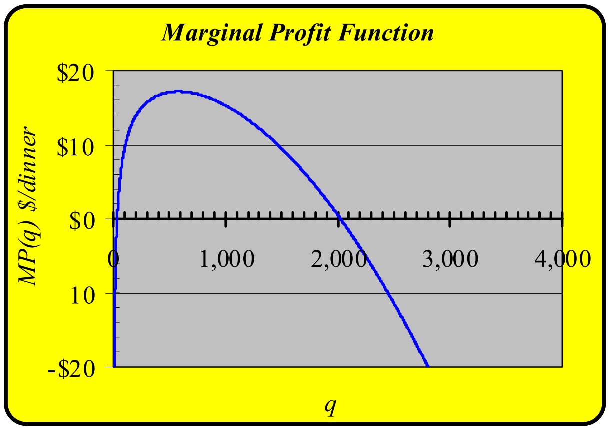
4. Answer the following questions using the graphs of profit and marginal given below.

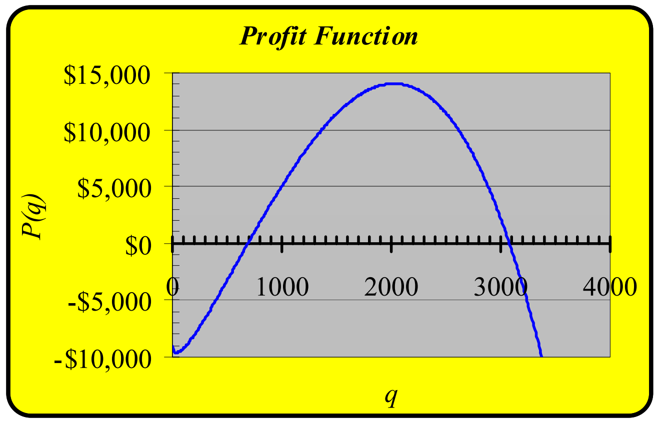
(a) Over what interval is $R(q) > C(q)$?

(b) Over what interval is $MR(q) > MC(q)$?

(c) For what quantity q does $MR(q) = MC(q)$?

(d) At what quantity q is the profit maximized?





5. A company manufactures and sells a special type of watch. Suppose the demand function is $D(q) = 56e^{-0.012q}$ measured in dollars with q measured in watches. Assume the function is only valid for $0 \le q \le 200$.

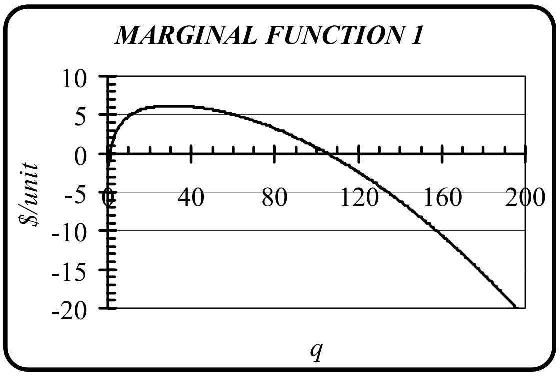
(a) Find $D(50)$ and give a business interpretation of your answer.

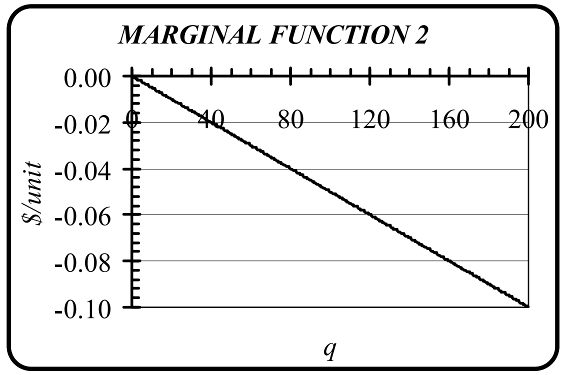
(b) If the company sells the watch for $D(50)$, write an expression to find the potential revenue lost because $D(50)$ is too high.

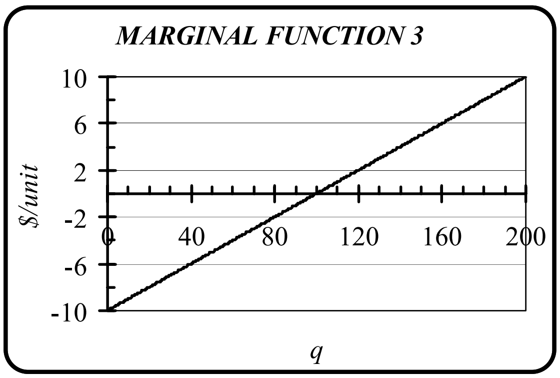
(c) Estimate the number of watches sold when the price of the watch is $40.

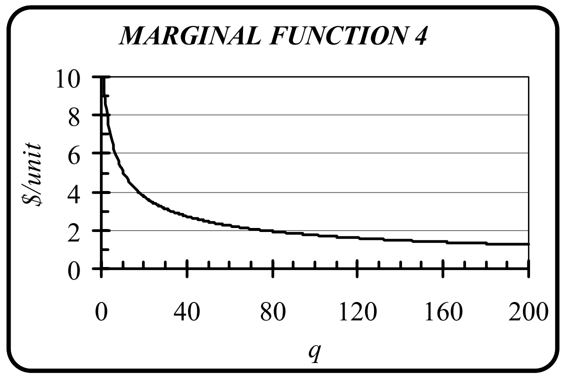
(d) Sketch a graph of $D(q)$. Give an illustration of $R(20)$.

6. The plots of four marginal functions are shown below.









A. Marginal Function ___ is marginal demand.

B. Marginal Function ___ is marginal cost.

C. Marginal Function ___ is marginal profit.

D. At what production level, q, are the variable costs equal to $2 per unit?

7. The demand function for a product is $D(q) = -2q^2 + 60$. Use a difference quotient with $h = 0.001$ to estimate the marginal demand when 5 items are produced. SHOW WORK AND GIVE ANSWER TO THREE DECIMAL PLACES.

8. If $f'(q) = m$, where $m \neq 0$ is a constant, what does this tell you about the graph of $f(q)$?

9. Let ***f(x)*** and ***g(x)*** be differentiable at x = -2.

Suppose $\boldsymbol{f}(-2)=3$, $\boldsymbol{g}(-2)=10$, $\boldsymbol{f}'(-2)=-4$, $\boldsymbol{g}'(-2)=-1$.

Consider the functions $h(x)=2f(x)-3g(x)$, $k(x)=5x+g(x)$, $l(x)=f(x)-10$

Evaluate A. $h'(-2)$ B. $k'(-2)$ C. $l'(-2)$ D. $h(-2)$

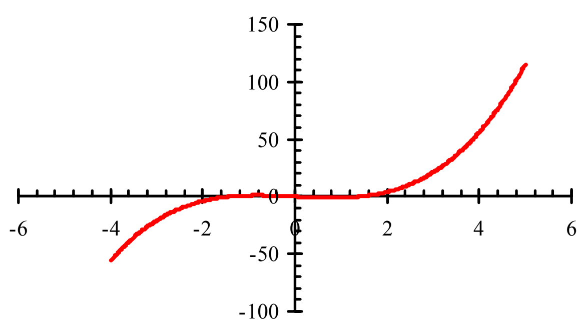
10. Let $f(x)=\dfrac{x^2}{50+x}$.

A. Use a difference quotient, with an increment of $h=0.001$ to estimate $f'(20)$.

B. Use the result from A. to find the equation of the line that is tangent to the graph of $f(x)=\dfrac{x^2}{50+x}$ at $x=20$.

C. Let $g(x)=\dfrac{-30*f(x)+85}{25}$. Use the result from A. to estimate $g'(20)$.

11. Let $f(x)=x^3-2x$. You are to approximate the area between the graph of f and the x-axis on the interval [-4, 5]. The graph is given below:



A. Find the points x_0, x_1, x_2, x_3 that divide the interval [-4, 5] into three subintervals.

B. Find the midpoints of the three subintervals m_1, m_2, m_3.

C. Find the midpoint sum $S_3(f, [-4, 5])$.

12. Your company has invented a new improved red rubber clown nose (these have better fit, ventilation, and longer lasting color). The fixed costs of production total \$11,000. Each clown nose costs an additional \$5 for materials and labor.

(a) Find the formula for the total cost function $C(q)$, where q is the number of clown noses produced.

(b) Find the formula for the marginal cost, $MC(q)$.

(c) The marginal revenue is given by $MR(q) = -0.02q + 150$. Use this and your answer to part B to find the quantity q which maximizes the profit.

13. The following questions refer to a good whose demand function is $D(q) = 200 - 0.2 \cdot q$. The fixed cost for producing the good is \$20,000 and it costs \$50 to produce each unit of the good.

(a) What price should we put on a unit, if we want to sell 600 units?

(b) How many units can we expect to sell at a price of \$120?

(c) What is the maximum price at which any unit of the good can be sold?

(d) Find an equation for the revenue function of the good. What revenue would result from the sale of 600 units at the price that would produce exactly 600 sales?

(e) Find an equation for the total cost function of the good. What is the total cost of producing 600 units?

(f) How many units of the good can be produce for a total cost of \$35,000?

(g) Find an equation for the profit function of the good. What profit would result from the sale of 500 units at the price which would produce exactly 500 sales?

(h) Suppose you know that the marginal profit is given by $MP(q) = 150 - 0.4 \cdot q$. Use this to find the number of units that should be sold in order to maximize profit.

(i) How should the good be priced in order to maximize profit? What maximum profit can be expected from sales of the good?

(j) Use differentiation to compute the marginal revenue when 300 units are being sold.

14. The marginal revenue and cost functions for a good are $MR(q) = 500 - 0.4q$ and $MC(q) = 250$ respectively.

(a) Is $R(q)$ increasing or decreasing at $q = 600$ units? Explain.

(b) Is $C(q)$ increasing or decreasing at $q = 1000$ units? Explain.

(c) Is $P(q)$ increasing or decreasing at $q = 800$ units? Explain.

(d) How many units should be produced and sold in order to maximize revenue?

(e) How many units should be produced and sold in order to maximize profit?

15. If $MC(q) = -0.04q + 120$, the cost of producing an extra item when 80 items have been produced is equal to________.

16. Each of the following statements implies one or more of the listed algebraic equations. In the space next to the statement, list the letter(s) of all corresponding equations

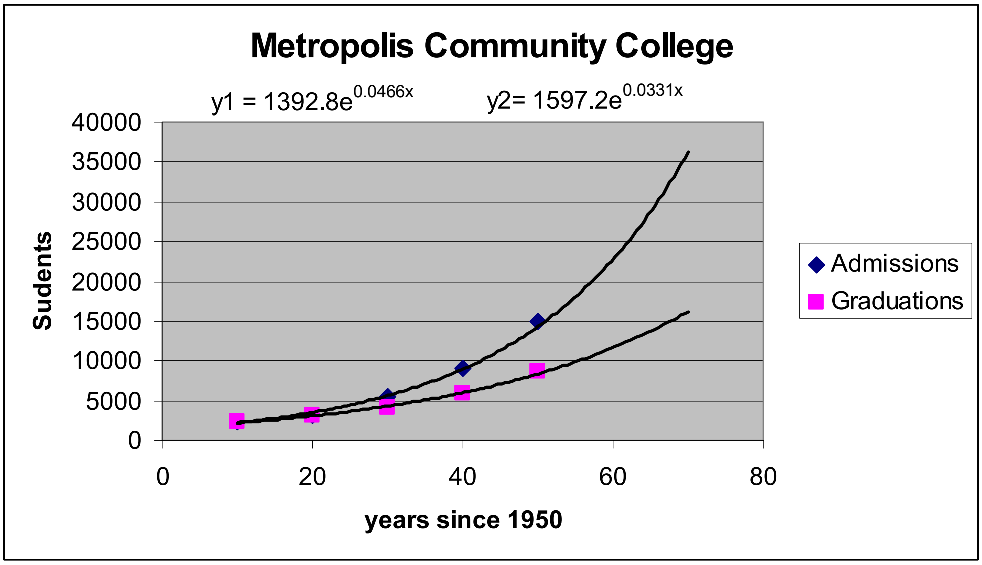
i) Profit is maximized when 500 units are produced.______________

ii) Above a price of $500 no units are sold.____________

iii) 500 units is a break-even point. __________

A) $P(500) = 0$
B) $MP(0) = 500$
C) $MR(500) = MC(500)$
D) $D(0) = 500$
E) $D(500) = 0$
F) $R(500) = 0$
G) $R(500) - C(500) = 0$
H) $P(q) = 500$

17. The chart below was produced for the regents of Metropolis Community College. It was generated using Excel to plot and generate exponential trend lines for the number of students admitted and the number of students graduating from MCC.



a) Based on the fitted functions shown for admissions and for graduations, estimate the number of admissions in 1965, and the number of students who graduated in 1965.

b) Estimate the number of students that will graduate in 2010.

c) The regents of MCC are very concerned with a trend that this chart shows between 1960 and 2000. Why?

d) Use the fitted functions to estimate the number of admissions in the year 2050. Should the regents make plans based on that number?

18. The fixed costs for a particular good are \$25000. It costs \$130 to produce the first 700 units of the good and it costs \$95 to produce any unit after that.

Enter below the information you will use in Graphing.xls to graph the cost function.

Definition

Formula for $f(x)$
=

Computation

x	$f(x)$
	=

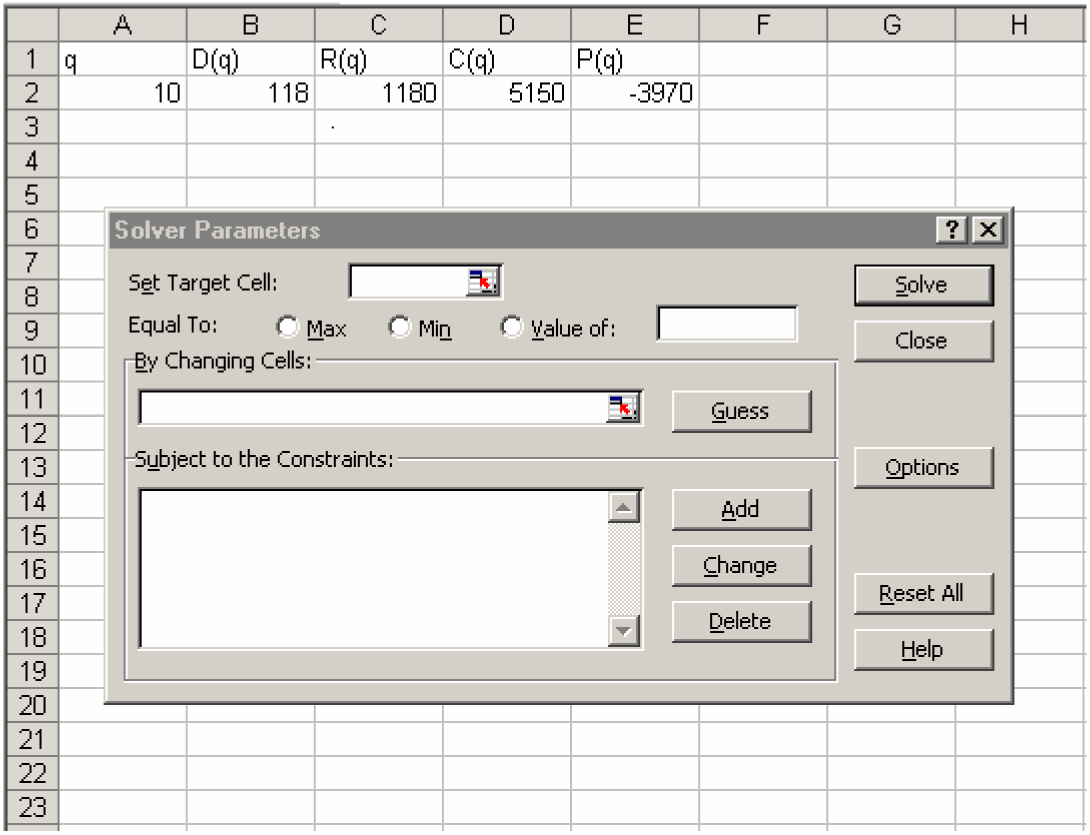
Plot Interval

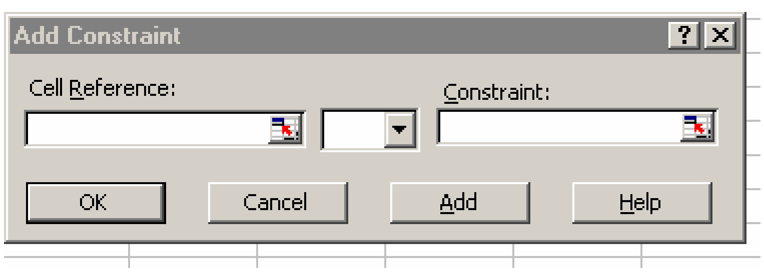
a	b

Constants

s	
t	
u	
v	
w	

19. Fill in the boxes in the screen capture below so that Solver will find the value of q that will lead to a profit of $2000 subject to the constraint that the Cost, $C(q)$ is positive.





20. This problem refers to Project 2. The table below shows a sample of signals from several companies from two previous auctions. All amounts are in millions of dollars.

Lease		Signals		
Auction	Value	Company 1	Company 2	Company 3
1	$55	$52	$53	$54
2	$120	$113	$127	$120

A. What is a signal?

B. Complete the following table:

Errors		
Company 1	Company 2	Company 3

C. Find the mean and standard deviation of the errors in part B.

D. In theory, what should the mean of the errors be? And why?

21. Explain the 5 random variable used in the oil lease project: V, S_V, R, C, and B.

22. What assumption in the oil lease project allows you to assume that $E(R) = 0$?

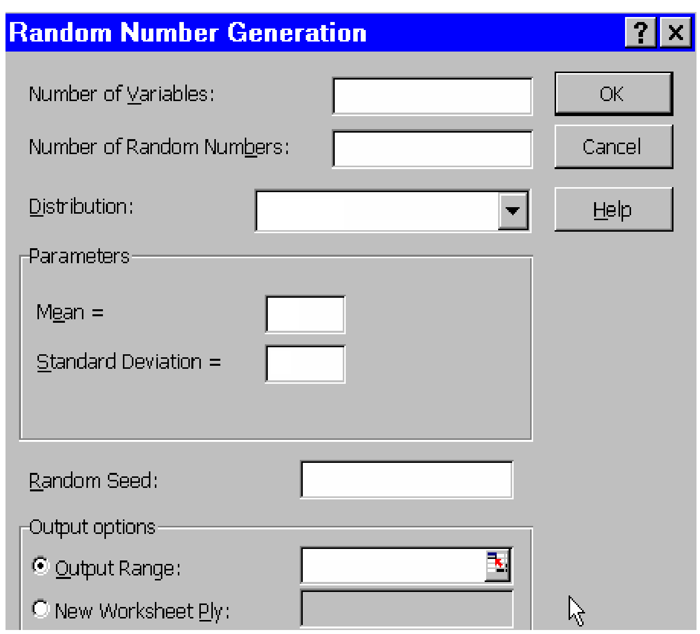
23. Approximately 62% of students who enroll in MATH 115B have a 3.3 grade point average or higher, the rest do not. Let G be the binomial random variable that gives the number of MATH 115B students with a 3.3 grade point average or higher. Assume that there are 700 students that enrolled in MATH 115B this year.

A. Find $E(G)$

B. Find $V(G)$

C. Find σ_G

24. Let X be a normal random variable with $\mu_X = 24$ and $\sigma_X = 3.2$. Fill in the information that would be needed to have the *Excel* function ***Random Number Generation*** create random values of X in ***Cells A1:F10***.



25. Suppose X is a finite random variable with the following probability mass function.

x	2	4	6	3
$f_X(x)$	0.4	0.2	0.1	A

(a) Find a value of A that makes this a valid probability model.

(b) Find the mean for this random variable.

(c) Find the standard deviation for this random variable.

26. It is claimed that 76% of the people in Tucson have at least one credit card. You plan to randomly select 5 people in Tucson and ask them if they have at least one credit card. Let Y be the number of people who say yes.

(a) What type of random variable is Y? Circle all that apply.

Discrete Continuous Uniform Binomial Exponential Normal

(b) What is the expected number of people who should say yes?

(c) Use the following table of the cumulative distribution function to find $P(Y = 3)$.

y	0	1	2	3	4	5
$F_Y(y)$	0.0008	0.0134	0.09325	0.34610	0.74645	1.000

27. Suppose X is a continuous random variable with the following probability density function:

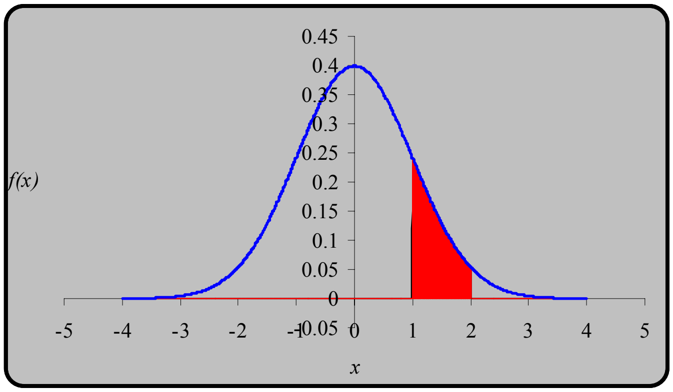
$$f_X(x) = \begin{cases} 0 & x < -1 \\ \dfrac{x^2}{3} & -1 \le x \le 2 \\ 0 & x > 2 \end{cases}$$

A. Set up an integral to find $E(X)$. Do NOT approximate the integral.

B. If $E(X) = 1.25$, set up an integral to find $V(X)$. Do NOT approximate the integral.

C. Set up an integral to find $P(X \le 1)$. Do NOT approximate the integral.

28. The graph of the p.d.f. for the standard normal random variable is given below. If the shaded area is 0.1359, find $P(-2 \le Z \le -1)$



29. Let $f(x) = \begin{cases} 0.2 & 1 \le x \le 6 \\ 0 & x < 1, x > 6 \end{cases}$

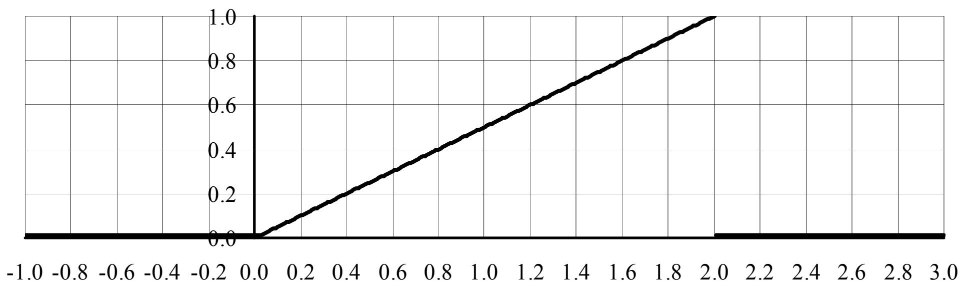
(a) Find $\int_2^5 f(x)dx$.

(b) Find $\int_4^9 f(x)dx$

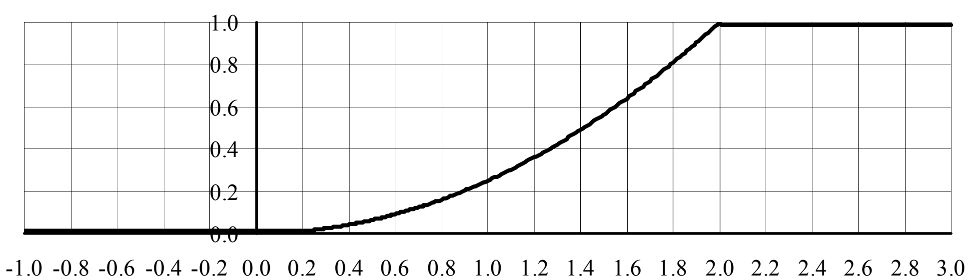
(c) If $f(x)$ is a probability density function, what does your answer in part A. represent?

30. Consider the continuous random variable, *X*. Use the graphs below to answer the following:

Plot A



Plot B



A. Determine which of the plots above is the p.d.f. of X and which is the c.d.f. of X?

B. On the plot of f_X, shade the region whose area corresponds to $P(0.8 \le X \le 1.6)$.
Estimate $P(0.8 \le X \le 1.6)$.

C. Use the plot of the *c.d.f.* of X to estimate $P(1.4 \le X \le 1.8)$.

31. Match the expressions on the left with their values on the right. A value may be used more than once.

A. $\int_0^\infty \frac{1}{5} e^{-x/5} dx$

B. $\int_0^\infty x \cdot \frac{1}{5} e^{-x/5} dx$

C. $\int_0^\infty (x-5)^2 \cdot \frac{1}{5} e^{-x/5} dx$

D. $\int_{-1}^4 0.2 dx$

a. 5

b. .25

c. 1

d. 1.5

e. 0.2

32. Let M be the normal random variable that gives the starting salary for a graduate from the school of business. Assume that $\mu_M = \$38{,}142$ and $\sigma_M = \$6{,}595$.

D. Give the expression for the probability density function of M.

E. If the 85% confidence interval for a standard normal random variable is (-1.44, 1.44), find a 85% confidence interval for M.

33. The mean travel time in New York State is 28 minutes. Let X be the time, in minutes, that it takes a randomly selected New Yorker to get to work on a randomly selected day. The travel times are normally distributed with a standard deviation of 9 minutes.

(a) Write the formula for the probability density function for X.

(b) Sketch the graph of the p.d.f.

(c) Set up an integral that gives the probability that the travel time for a randomly selected New Yorker is at least 37 minutes.

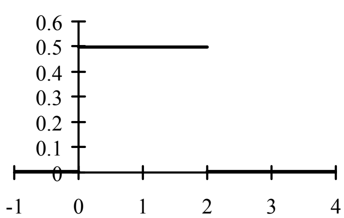
(d) Represent the integral above on the graph of the p.d.f.

(e) Give the exact Excel function that would be used to calculate $P(X \leq 30)$.

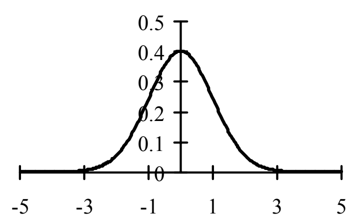
34. Let X be the continuous random variable, that is exponential with $E(X) = 3$, and let S be the standardization of X. Compute $P(0 \leq S \leq 4)$.

35. Consider the following plots of *p.d.f.'s*.

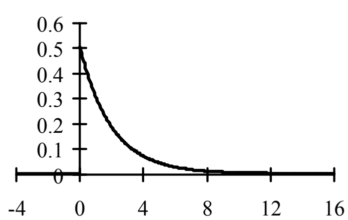
a.



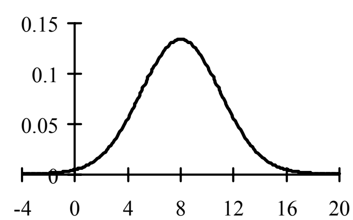
b.



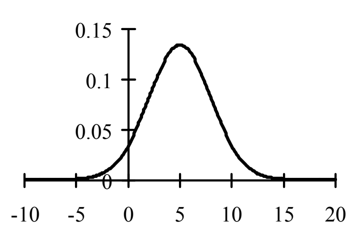
c.



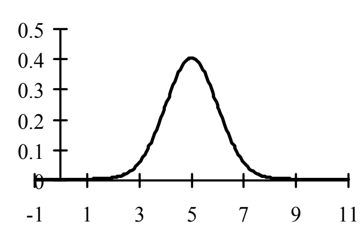
d.



e.



f.



A. Which one could correspond to a standard normal random variable?

B. Which one could correspond to a uniform random variable?

C. Which one could correspond to an exponential random variable?

D. Which ones could ***not*** possibly correspond to a normal random variable?
(There might be more than one.)

E. Which one could correspond to a normal random variable with $\mu_X = 5$ and $\sigma_X = 3$?

36. X can assume only the values of 0 and 1, with $P(X = 0) = 0.7$ and $P(X = 1) = 0.3$. Let S_2 be the standardization of the sample mean for X, with sample sizes of $n = 2$.

(i) Compute the mean and standard deviation of $\bar{x}$.

(ii) Compute all values for the *p.m.f.* of S_2.

(iii) Compute the mean and standard deviation of S_2.

37. Consider the following sample of size $n = 6$. 10.3, 12.4, 8.9, 10.3, 9.0, 11.8

A. Compute the sample mean, $\bar{x}$

B. Compute the sample variance, s^2.

C. Compute the sample standard deviation, s. Round your answer to 3 decimal places.

38. Let X be a random variable with the probability mass function at the right.

x	0	1	2	3	4
$f(x)$	0.4096	0.4096	0.1536	0.0256	0.0016

A. Find μ_X and σ_X .

B. Now suppose the following sample of seven observations was made of the random variable X : 4, 3, 1, 1, 0, 1, 4. Find the mean and standard deviation of this sample.

C. Consider the sample mean, $\bar{x}$, to be a random variable. Find the mean and standard deviation of this random variable.

39. Let X be the random variable that gives the time it takes for an employee of a company to learn a new task. It has been determined that the mean of X, μ_X, is 6 and the standard deviation, σ_X, is 1.6. A random sample of 50 employees is taken. Let $\bar{x}$ be the random variable that is the mean of such a random sample.

(a) Find $\mu_{\bar{x}}$

(b) Find $\sigma_{\bar{x}}$

(c) Give a formula for the random variable, S, that is the standardization of $\bar{x}$.

(d) What is the approximate distribution of S?

40. The scores on the Math 115b test were approximately normally distributed.
Your friend scores 68 points on the Math 115b test. He wants to assure his advisor that this is a decent score based on the scores of the other students. So, he asks 5 friends in class what they scored on this test. He calculates that the mean of these 5 scores is 72.4 and that the standard deviation is 8.2.

Using the expression below, calculate the 95% confidence interval for the mean of all scores. For this confidence level, $z_0 = 1.96$.

$$\left(\bar{x} - z_0 \cdot \frac{\sigma_x}{\sqrt{n}}, \quad \bar{x} + z_0 \cdot \frac{\sigma_x}{\sqrt{n}}\right).$$

41. Let V be the random variable that gives the proven value of an oil lease similar to the one which is to be auctioned. A random sample of 36 historical proven values are used to estimate the mean proven value of V. The sample mean is \$93 million and its standard deviation is \$13.4 million. Use the information above to find a 90% confidence interval for the mean proven value of all leases and give a practical interpretation.
For this confidence level, $z_0 = 1.645$.

$$\left(\bar{x} - z_0 \cdot \frac{\sigma_x}{\sqrt{n}}, \quad \bar{x} + z_0 \cdot \frac{\sigma_x}{\sqrt{n}}\right).$$

42. Let X be a random variable that is uniform over the interval $[0,5]$.

(i) Find $f_X(x)$.

(ii) Find an expression for the following.

$$\int_0^x f_X(u)\,du =$$

(iii) Differentiate the function you found in (ii).

43. The cost of producing a new type of sunglasses is given by $C(q) = 40{,}000 + 70q$, the marginal revenue is $MR(q) = -0.001q + 150$.

(i) What is the quantity that maximizes the profit?

(ii) The company invested in some equipment innovation which resulted in the marginal cost to be reduced by 15%. The new equipment cost is $9,000.

Find the new quantity of sunglasses that would maximizes the profit.

44. Suppose that for a certain kind of product, revenue ***R*(1200)=$30,000**, cost ***C*(1200)=$23,000**, marginal revenue ***MR*(1200)=$400**, and marginal cost ***MC*(1200)=$100.** Because of changes in economy, the revenue function decreased by $5000, and cost increased by 10%. Find the profit and the marginal profit of producing 1200 items under new economic conditions.

45. Find integral $\int_0^x 0.2e^{-0.2u}\,du$

46. Find the derivative of $F(x) = \begin{cases} 1 - e^{-x/15} & \text{for } x \ge 0 \\ 0 & \text{for } x < 0 \end{cases}$

47. Let the random variable X represents time (in hours and parts of hours) needed for a business student to complete the final exam. The data is collected for the sample of 90. Out of 90 students, the sample mean was $\bar{x} = 1.6$, and the sample standard deviation $s = 0.2$. Find the 95% confidence interval for the mean of X, given $z_0 = 1.96$.

48. Two companies are bidding against each other where the outcome of the bidding is determined by the 3 x 3 matrix shown below. Each company is trying to maximize its gain and is required to bid if it improves its current situation. Company 2's payoff (in millions of dollars) is shown in the top right of each square and Company 1's payoff is shown at the bottom left. Company 1 bids first and selects a row, then Company 2 bids by selecting a column. The amount in the square at the intersection of the selected row and column is the potential pay off. Suppose for example Company 1 chooses row 3 (which contains a maximum pay off for Company 1 of $9 million), then Company 2 would choose column A (which contains its maximum pay off for this row). If bidding stopped at this point Company 1 would receive $4 million and Company 2 would receive $9 million. But Company 1 may now change its row selection, and Company 2 may then change its column selection. Bidding continues until both companies settle on a single row and column. If each company always bids to improve its situation then this point is a Nash Equilibrium. Note: a Nash Equilibrium is not necessarily the best outcome for both companies, and there can be more than one Nash Equilibrium.

		Company 2		
		A	B	C
Company 1	1	1 7	3 8	6 5
	2	2 3	4 1	5 6
	3	9 4	8 9	7 2

a) Which square above is a Nash Equilibrium, and why?