Research Statement

Kristen A. Beck

The field of commutative algebra, at its core, is the theory of characterizing solutions to systems of polynomial equations. Commutative rings and their modules are often studied through the utility of powerful homological methods, wherein each finitely generated module is essentially replaced by a (possibly infinite) sequence of homomorphisms of free modules, called a *free resolution.* The functionality of this approach lies in the facts that (1) any module can be 'resolved' in a minimal sort of way, and (2) all of the important characteristics of a minimal free resolution are well-defined over a local ring; many of them are even well-defined in the more general setting. For example, over the polynomial ring $R = k[x, y, z]$, the module $k \cong R/(x, y, z)$ is minimally resolved by the following Koszul complex.

$$0 \to R \xrightarrow{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} R^3 \xrightarrow{\begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}} R^3 \xrightarrow{[x\ y\ z]} R \to k \to 0$$

Furthermore, given a minimal free resolution, one can explicitly read off the relations among the generators of the module, as well as the relations among these relations, and so on. In this way, a minimal free resolution quantifies a module's deviation from being free. In fact, this characterization is just the tip of the iceberg; it turns out that there is a wealth of data hidden in free resolutions.

Broadly speaking, the bulk of my research regards the study of infinite free resolutions over a commutative local (Noetherian) ring. Such resolutions are fascinating in the sense that they have not been completely characterized in general. As one might expect, gleaning information about a module from its (minimal) infinite resolution often goes hand-in-hand with understanding the long-term behavior of certain invariants of the resolution. My research to this end investigates the asymptotic behavior of the *syzygies* — that is, relations — in a minimal resolution. It has been shown [19] that there is a strong connection between the algebraic structure of (high) syzygies of the residue field of a local ring and that of the ring itself. Micah Leamer and I extend these ideas in [8] by studying the Krull dimension of high syzygies in certain infinite resolutions. The motivation for this work and the related results are discussed more thoroughly in Section 1.1.

My other focus in this area concerns the existence of doubly-infinite resolutions which enjoy a sort of reflexivity property. Modules possessing this type of resolution are called *totally reflexive* because they are not only reflexive in the classical sense, but in a homological sense as well. Such modules are used to characterize Gorenstein rings — that is, rings which have finite injective dimension over themselves — and generalize projective dimension. Among other niceties, the existence of totally reflexive modules makes it possible for one to study the questions of homological algebra in a broader setting: by using resolutions by totally reflexive modules in place of projective (resp. free) resolutions. However, the collection of rings which admit non-trivial totally reflexive modules has not yet been completely classified. My work toward understanding the existence of these modules has taken two directions: in [5] I study the existence of totally reflexive modules induced by a Gorenstein homomorphism, and in [6] I investigate a special class of totally reflexive modules defined over a local ring having the property that the fourth power of its maximal ideal is zero. Section 1.2 details this work.

A second, seemingly disconnected, interest of mine pertains to differential graded (DG) algebra, whose main characters are DG algebras and their DG modules. Put simply, these gadgets can be viewed as graded algebras (over a commutative ring) and their modules, each equipped with a differential — that is, a degree -1 endomorphism with square zero — which 'cooperates' with the algebra's action. One motivation for studying differential graded algebra is rooted in the need for homological tools which are capable of handling complexes, rather than just modules; cf. [2]. This viewpoint is paramount inside the derived category, whose objects are themselves complexes. An interesting question to ask within the field of differential graded algebra regards the level of 'richness' of the category of DG modules over a DG algebra with trivial differential; specifically, how much does this category differ from that of the graded modules over a graded algebra? In [7], I utilize the so-called *totaling functor* to answer this question for the category of DG modules over a polynomial ring. Motivation and results concerning my work to this end can be found in Section 2.1. Another intriguing question in this field is whether certain invariants for complexes of modules can be extended in a natural way to the class of DG algebras. Sean Sather-Wagstaff and I recently explored this question with respect to Krull dimension; cf. [9]. Discussion and main results from this work can be found in Section 2.2.

Finally, I discuss in Section 3 my projected research trajectory by highlighting a few interesting open questions and their connections to the literature. Section 4 details my experience regarding the direction of undergraduate research and provides some examples for future project ideas to this end.

1. Infinite Free Resolutions

In this section, $(R, \mathfrak{m}, k)$ shall denote a commutative local (Noetherian) ring R with unique maximal ideal $\mathfrak{m}$ and residue field $k := R/\mathfrak{m}$. There will be no loss of generality in assuming R-modules to have infinite projective dimension.

1.1. **Dimensions of syzygy modules.** The nth *syzygy module* of a finitely generated R-module M, denoted Ω_n, is defined to be the cokernel of the nth map in a minimal R-free resolution of M. It is known that the syzygies of the residue field of a local ring $(R, \mathfrak{m}, k)$ encode information about the structure of the ring itself. In particular, (1) R is regular if and only if all high syzygies of k are free, (2) R is Gorenstein if and only if its extension by any high syzygy of k is trivial, and (3) R is Cohen-Macaulay if and only if all high (nonzero) syzygies of k are also Cohen-Macaulay [19]. Given these classifications, it is natural to wonder whether the high syzygies of an *arbitrary* finitely generated module also carry information about the ring. Indeed, in [19] Okiyama demonstrates that the depths of such (infinite) syzygies eventually stabilize to that of the ring. The motivation for my work with Micah Leamer in [8] is to consider whether a similar statement can be made regarding a far less algebraic invariant — namely, dimension. The driving force behind our results lies in the fact that all syzygies of an R-module with strictly increasing Betti numbers have dimension equal to that of the ring. In [8], we show that one does not need such a strong condition on the Betti numbers of a module in order to glean information about the supports of its syzygies.

1.1.1. **Theorem.** *Let M be a finitely generated module over a local ring R. If the Betti numbers of M are eventually non-decreasing, then the sequences $\{\dim \Omega_{2i}\}_{i\geq 0}$ and $\{\dim \Omega_{2i+1}\}_{i\geq 0}$ are both eventually constant. If $\operatorname{pd}_R M = \infty$, then one sequence stabilizes to $\dim R$, and the other stabilizes to $\dim R/\mathfrak{p}$ for some minimal $\mathfrak{p} \in \operatorname{Supp} M$.*

As an immediate corollary to Theorem 1.1.1, all high syzygies of a module with eventually non-decreasing Betti numbers over an equidimensional local ring will have dimension equal to that of the ring. Examples of such rings include domains and rings with dimension at most one.

1.2. **Existence of totally reflexive modules.** A finitely generated R-module M is called *totally reflexive* if it is reflexive, and if the minimal free resolutions of M and $M^* := \operatorname{Hom}_R(M, R)$ remain exact upon dualization — that is, upon application of $\operatorname{Hom}_R(-, R)$. The latter criterion is equivalent the vanishing of the cohomology functors $\operatorname{Ext}^i_R(M, R)$ and $\operatorname{Ext}^i_R(M^*, R)$, respectively, for all $i > 0$. As a direct result, every totally reflexive R-module admits a 'doubly-infinite' minimal free resolution, called a *complete resolution*, which possesses a sort of reflexivity condition.

Every ring admits totally reflexive modules in the trivial (i.e. projective) sense. Furthermore, over a Gorenstein local ring, the maximal Cohen-Macaulay modules and totally reflexive modules coincide. It is away from this landscape of Gorenstein rings that non-trivial totally reflexive modules are more difficult to pin down. However, recent studies [11, 12, 15, 20] have shown that, in many cases, the existence of a single non-trivial totally reflexive module guarantees the existence of infinitely many such modules, a fact that points to the sufficiency and importance of characterizing rings which admit *at least* one non-trivial totally reflexive module module. This is precisely the direction that my research in this area takes.

In [3], Avramov, Gasharov, and Peeva prove that every embedded deformation ring admits non-trivial totally reflexive modules. Such a ring, which can be realized as the quotient of a local ring by a regular sequence contained in the square of the maximal ideal, is not always Gorenstein. To generalize this result, I consider in [5] the class of local rings of the form $R = S/I$, where I is a Gorenstein ideal.

1.2.1. **Theorem.** *Let $\varphi \colon Q \to R$ be a Gorenstein homomorphism of local rings whose kernel is contained in $\mathfrak{m}_Q^2$. Suppose that there exists a Gorenstein local ring P and homomorphism $\psi \colon P \to Q$ of finite flat dimension. Furthermore, let $S = P/I$ be a lifting of R to P via Q such that $I \subseteq \mathfrak{m}_P^2$ and $\operatorname{grade}_P(S, P) \geq \operatorname{grade}_P(S, Q)$. Then R admits non-trivial totally reflexive modules.*

Among its corollaries [5, 2.6, 2.8, 2.10], one nice asset of Theorem 1.2.1 is that it recovers [3, Theorem 3.2]. Further, while the authors of [3] provide an explicit construction of totally reflexive modules over a ring with an embedded deformation, my proof of the same result constructs ring homomorphisms which satisfy the hypotheses of the above theorem. Finally, in [5, Construction 3.1] I provide a simple construction for (non-Gorenstein) rings which fit the hypotheses of Theorem 1.2.1, but do not have embedded deformations. As it turns out, the 'smallest' such explicit example illustrating this construction has codimension nine [5, Example 3.2].

In [21], Yoshino examines the structure of a local ring $(R, \mathfrak{m}, k)$, satisfying $\mathfrak{m}^3 = 0$, which admits a non-trivial totally reflexive module. The author demonstrates that whenever such a ring is non-Gorenstein, (1) its Hilbert polynomial[1] must have the form $H_R(t) = 1 + et + (e-1)t^2$, where $e := \dim_k \mathfrak{m}/\mathfrak{m}^2$ is the embedding dimension of R, and (2) the Betti numbers of the module must be constant. One cannot expect this characterization to completely extend to the class of local rings $(R, \mathfrak{m})$ for which $\mathfrak{m}^4 = 0$; indeed, there exist many non-projective totally reflexive modules over such rings whose Betti sequences are non-constant. The motivation for my work in [6] is to determine exactly how much of Yoshino's characterization *does* extend to the $\mathfrak{m}^4 = 0$ case. Assuming that such a ring admits a non-trivial totally reflexive module, I ask two questions: (1) is the Hilbert polynomial of R balanced (i.e. is $H_R(-1) = 0$)? and (2) what is the possible growth of the ranks of the free modules in the complete resolution of the module? My results in [6] focus on R-modules M with eventually linear minimal free resolutions.

1.2.2. **Lemma.** *Let M be a finitely generated module with an eventually linear minimal free resolution over a local ring $(R, \mathfrak{m})$ satisfying $\mathfrak{m}^4 = 0$, and suppose that the Betti sequence of M is not eventually constant. If $\operatorname{Ext}^i_R(M, R)$ vanishes for all $i \gg 0$, then the Hilbert polynomial of R can be expressed in terms of the embedding dimension of R and four sufficiently high Betti numbers of M.*

Making further assumptions regarding the growth rate of the Betti sequence of M, one can say even more. One rate which turns out to be particularly important is called *exceptional* — specifically, a Betti sequence $\{b_i\}$ is exceptional if $b_{i+3} + b_i = b_{i+2} + b_{i+1}$ for $i \gg 0$.

1.2.3. **Theorem.** *Let M be a finitely generated module with an eventually linear minimal free resolution over a local ring $(R, \mathfrak{m})$ satisfying $\mathfrak{m}^4 = 0$. If $\operatorname{Ext}^i_R(M, R)$ vanishes for all $i \gg 0$, the following hold.*

(1) *If the Betti sequence of M has non-exceptional polynomial growth, then $H_R(t) = 1 + et + et^2 + t^3$.*
(2) *If the Betti sequence of M has exponential growth of base a, then $H_R(t) = 1 + et + ft^2 + gt^3$, where*

$$f = \left(a + \frac{1}{a}\right) e - \left(a^2 + 1 + \frac{1}{a^2}\right) \qquad \text{and} \qquad g = e - \left(a + \frac{1}{a}\right).$$

An immediate corollary to Theorem 1.2.3(2) is that the quantity a must be irrational. Moreover, note that for appropriate values of a, the respective Hilbert polynomial is not balanced. One such explicit example is given in [6, Example 4.2.13]. By considering the consequences of a particular ring admitting two modules with differing growth in their Betti sequences, I show [6, Theorem 4.3.3] that certain asymmetric exponential growth is not possible over an $\mathfrak{m}^4 = 0$ local ring. Furthermore, in [6, Theorem 4.3.1], I demonstrate that certain polynomial-exponential growth is only possible over an $\mathfrak{m}^4 = 0$ local ring whenever the ring has a symmetric Hilbert polynomial. Indeed, the existence of such a complete resolution has already been established; cf. [16].

2. Differential Graded Algebra

Throughout this section, R shall denote a commutative Noetherian ring with identity. A complex A of R-modules is a *DG algebra* if it is endowed with an associative product which agrees with the differential of A in such a way that the Leibniz rule is satisfied. Furthermore, if X is a complex of R-modules which has the structure of a (graded left) A-module in such a way that the action of A on X also satisfies the Leibniz rule, then X is called a *DG A-module*. These 'product rules' are summarized as follows.

$$\partial^A(ab) = \partial^A(a)b + (-1)^{|a|} a \partial^A(b)$$
$$\partial^X(ax) = \partial^A(a)x + (-1)^{|a|} a \partial^X(x)$$

2.1. **The totaling functor.** By examining the above Leibniz rules, one can see that a DG module over an arbitrary DG algebra is certainly a much more complex gadget than a mere complex of graded modules. Indeed, the former case not only requires that both the complex and the algebra be equipped with differentials, but also that these differentials cooperate. However, if one strips away the differential of a DG algebra A, so that A is defined by its underlying graded algebra $A^\natural$, how 'different' is X from a complex of (graded left) $A^\natural$-modules? This question was asked by Avramov and Jorgensen in [4], and is the motivation for my work in [7].

My approach to this question is involves an investigation of the *totaling functor* Tot, from the category $\mathbf{ChG}(A)$ of complexes of graded $A^\natural$-modules to the category $\mathsf{DG}(A)$ of DG A-modules. The action of Tot is defined as follows: given a complex X of graded $A = A^\natural$-modules, the underlying graded structure of $\operatorname{Tot} X$ is given by

$$(\operatorname{Tot} X)^\natural = \bigoplus_{i \in \mathbb{Z}} \Sigma^i X_i$$

[1]This is a slight abuse of notation. H_R is intended to actually represent the Hilbert polynomial of the associated graded ring of R. However, as the Hilbert polynomial is only defined for graded objects, this convention should not be a cause for confusion.

where Σ^i denotes the ith (degree) shift, and the differential of $\operatorname{Tot} X$ is inherited in a natural way from the differential on X; cf. [7]. In order to ensure that the structure obtained from the totaling of a complex is indeed a DG module, it is necessary to somehow impose the Leibniz rule; this is achieved by defining scalar multiplication to behave as a chain map.

My work in [7] seeks to determine whether $\operatorname{Tot}\colon \mathbf{ChG}(A) \to \mathsf{DG}(A)$ is an onto functor when A has a trivial differential; with an affirmative answer, there would be no loss of generality in viewing a DG module over a DG algebra $A = A^\natural$ as a complex of graded A-modules. My main result in [7] is established for perhaps the simplest graded algebra: a polynomial ring over an arbitrary field.

2.1.1. **Theorem.** *Let A be a polynomial ring in more than one variable over a field. Then the totaling functor $\operatorname{Tot}\colon \mathbf{ChG}(A) \to \mathsf{DG}(A)$ is not onto.*

It turns out that Theorem 2.1.1 does not extend more generally to the class of all polynomial rings over a field. Indeed, I show in [7, Theorem 2.11] that every DG module over a polynomial ring in one variable is quasiisomorphic to the totaling of some complex of graded modules. This constructive proof, made possible by the structure theorem for finitely generated modules over a principal ideal domain, implies that the totaling functor defined on the derived categories is onto.

2.2. **Systems of parameters and Krull dimension.** In 1979, Foxby became the first to formalize the notion of Krull dimension for a complex of R-modules [13]. His definition is stated in terms of lengths of chains of graded prime ideals in $\operatorname{Supp}_R X = \bigcup_{i\in\mathbb{Z}} \operatorname{Supp} \mathrm{H}_i(X)$.

$$\dim_R X = \sup\{\dim R/\mathfrak{p} - \inf X_\mathfrak{p} \mid \mathfrak{p} \in \operatorname{Supp}_R X\}$$

According to Foxby's definition, it is possible for an R-complex to have negative dimension. Still, for complexes with non-trivial infimum (i.e. greater than $-\infty$), one can see that dimension is indeed bounded below. This fact provides a notion for 'minimal' prime ideals, which Christensen refers to as anchor primes in [10].

When $R = (R, \mathfrak{m})$ is local, it is natural to seek a definition for a system of parameters for a (homologically finite) R-complex X which agrees with Foxby's definition of dimension. Christensen developed one such notion in [10], definin a sequence $\mathbf{x} \in \mathfrak{m}$ to be a *system of parameters* for X if and only if $\mathfrak{m}$ is an anchor prime for each $\mathrm{H}_i(K^R(\mathbf{x}) \otimes_R X)$. Two interesting questions arise: (1) can one develop an alternate definition for a system of parameters $\mathbf{x}$ for X which is characterized by the finite length of the R-modules $\mathrm{H}_i(K^R(\mathbf{x}) \otimes_R X)$, and (2) if so, is this notion equivalent to that formulated by Christensen? These questions provide the initial motivation for my work in [9] with Sean Sather-Wagstaff. A secondary motivation for our work is to compare the resulting notions of dimension for a homologically-finite Noetherian DG algebra.

The existence of a sequence $\mathbf{x} \in \mathfrak{m}$ such that each of the Koszul homology modules $\mathrm{H}_i(K^R(\mathbf{x}) \otimes_R X)$ has finite length over R is certainly guaranteed; indeed, any generating sequence for an $\mathfrak{m}$-primary ideal of R fits the bill. Whenever such a sequence $\mathbf{x}$ is of the shortest possible length, we call it a *length system of parameters* for X. Furthermore, given a length system of parameters $\mathbf{x} = x_1, \ldots, x_\ell$ for X, the *length dimension* of X is defined to be the quantity $\operatorname{ldim}_R(X) := \ell - \inf(X)$. As it turns out, our notion of (length) dimension does not always agree with that of Foxby, given above. While every length system of parameters is also a system of parameters, the converse is not true in general. However, our main result states that this phenomenon cannot occur when X admits the structure of a DG R-algebra.

2.2.1. **Theorem.** *Let A be a homologically finite DG A_0-algebra such that $(A_0, \mathfrak{m}_0)$ is Noetherian.*

(1) *Given a sequence $\mathbf{x} \in \mathfrak{m}_0$, the following are equivalent.*
   (a) $\mathbf{x}$ *is a system of parameters for A.*
   (b) $\mathbf{x}$ *is a length system of parameters for A.*
   (c) $\mathbf{x}$ *is a system of parameters for A_0.*

(2) $\operatorname{ldim}_{A_0} A = \dim_{A_0} A = \dim \mathrm{H}_0(A)$.

Moreover, we show that it is possible to define a concept of DG Krull dimension which, over certain DG algebras, agrees with the quantities given in Theorem 2.2.1(2) above.

3. Future Projects

I see my research path following two distinct trajectories. Broadly speaking, these are (1) studying properties of infinite free resolutions, and (2) extending notions from commutative algebra to differential graded commutative algebra. With respect to the first area, I am very interested in a long-standing open question of Avramov.

3.1. *Question.* [1] Is the Betti sequence of any finitely generated module over a local ring eventually non-decreasing?

This question has been given a positive answer for both Golod rings and complete intersections. Furthermore, Lescot also answers the question for a local ring with $\mathfrak{m}^3 = 0$ [17]. In light of the setting of my research in [6], it would be interesting to extend Lescot's result to the class of local rings for which $\mathfrak{m}^4 = 0$.

An even more specific query than that in Question 3.1 is concerned with the periodicity of Betti sequences. Indeed, there is no known example of a module whose Betti sequence is periodic, of period greater than one. In [6, Theorem 3.1.2] I establish a result which only calcifies this fact; namely, over a local $\mathfrak{m}^4 = 0$ ring R, a module M with an eventually linear partially complete minimal free resolution cannot have positive periodicity less than four. I would like to be able to extend this result.

3.2. *Question.* Let $(R, \mathfrak{m})$ be a local ring such that $\mathfrak{m}^n = 0$ for some positive integer n. If M is a finitely generated R-module which possesses an eventually linear partially complete minimal free resolution, can the Betti sequence of M be eventually periodic, of period $1 < m < n$?

Stemming from my recent results with Sean Sather-Wagstaff in [9], I am also interested in questions regarding differential graded (commutative) algebra. Specifically, given the notions for Krull dimension and systems of parameters for a DG algebra A, what classical invariants and concepts from commutative algebra can be extended to the DG setting? For example, Frankild and Jørgensen define a notion for a Gorenstein DG algebra in [14].

3.3. *Question.* Is it possible to define a notion for a Cohen-Macaulay DG algebra which would generalize a Gorenstein DG algebra, as defined in [14], in the expected way?

4. Undergraduate Research

While at the University of Arizona, I have directed two undergraduate research projects involving a total of four students. These are detailed below.

(Non-)Periodicity of Betti Numbers Over a Class of Short Local Rings. In this project, funded by the Western Alliance to Expand Student Opportunities, I mentored three minority mathematics majors during Summer 2013 in what turned out to be a windy road which ultimately strengthened the students' understanding of linear algebra. Put simply, the students learned how the most basic constructions of homological algebra (i.e. free resolutions) illustrate precisely how much 'niceness' is lost when generalizing from a vector space over a field to a module over a ring. Focusing on finitely generated modules over a polynomial ring (or its quotient by a principal ideal), the students gained an intuition for how relations between generators give rise to syzygy modules, which iteratively give rise to a free resolution.

Applications of the Yoneda Lemma. In this ongoing project, I am mentoring one computer science major in cross-discipline project which investigates the application of the (category theoretic) Yoneda Lemma [18, Ch. III, Sec. 2] to both group theory and computer science. The initial part of this project involves a diagrammatic 'proof without words' of the Yoneda Lemma, which nicely illustrates the fullness and faithfulness of the Yoneda embedding. (This project was preceded by a reading course in basic category theory, wherein the student learned the basic machinery needed for the project.) Furthermore, the student is currently working on understanding the applications of the Yoneda Lemma to propositional logic.

I would also like to involve undergraduate students in questions stemming from my research interests listed in Section 3. Specifically, a first attempt at answering Question 3.2 would only require a knowledge of basic linear algebra. Other requisite skills could easily be acquired during the course of the project.

In addition to supervising research projects, I also organize a popular bi-weekly 'brown bag' seminar whose goal is to introduce curious undergraduates to accessible research problems in a variety of mathematical fields; cf. `http://math.arizona.edu/~kbeck/brown-bag`.

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