(9) $A - \lambda I = \begin{bmatrix} -1-\lambda & 3 & 0 \\ 3 & -1-\lambda & 0 \\ -2 & -2 & 6-\lambda \end{bmatrix}$.

$$\det(A - \lambda I) = (6-\lambda)\left[(-1-\lambda)^2 - 9\right]$$
$$= (6-\lambda)\left[\lambda^2 + 2\lambda - 8\right]$$
$$= (6-\lambda)(\lambda+4)(\lambda-2).$$

The eigenvalues are $\lambda_1 = 6$
$\lambda_2 = 4$
$\lambda_3 = 2$.

Let's find $\mathcal{E}_6$. We have to find the nullspace of $A - 6I$.

$$A - 6I = \begin{bmatrix} -7 & 3 & 0 \\ 3 & -7 & 0 \\ -2 & -2 & 0 \end{bmatrix} \sim \begin{bmatrix} +1 & 1 & 0 \\ 3 & -7 & 0 \\ -7 & 3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & -10 & 0 \\ 0 & 10 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

$x_3 = s$.
$x_2 = 0$
$x_1 = 0$.

Hence $\mathcal{E}_6 = \text{span}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\mathcal{E}_4$ and $\mathcal{E}_2$ can be fund similarly.

(10) $A = \begin{bmatrix} 3 & 4 & 3 \\ 5 & 7 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, a) $\text{adj } A = \begin{bmatrix} 7 & -4 & -13 \\ -5 & 3 & 9 \\ 0 & 0 & 1 \end{bmatrix}$

b) $A^{-1} = \frac{1}{\det A} \cdot \text{adj } A = \text{adj } A$ since $\det A = 1$.

(11) a) -12

b) $a^3 + b^3 + c^3 - 3abc$.

(12) a) $\lambda_1 = \lambda_2 = 3 \quad \lambda_3 = 5$.

c) $v_1 = [1, -1, 0]$
$v_2 = [1, 0, 1]$
$v_3 = [1, 2, 1]$.

d) Yes. $P = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ $\quad D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$.

(13) $A^2 = \begin{bmatrix} 11 & 5 \\ 10 & 6 \end{bmatrix}$ $\quad A^3 = \begin{bmatrix} 43 & 21 \\ 42 & 22 \end{bmatrix}$

(14) Yes.

(15) $A = \begin{bmatrix} 3 & 0 & 0 & \ldots \end{bmatrix}$