Solutions

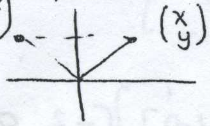
1(a) $T\begin{bmatrix} x_1+x_2 \\ y_1+y_2 \end{bmatrix} = \begin{bmatrix} 1+x_1+x_2 \\ y_1+y_2 \end{bmatrix} \neq \begin{bmatrix} 1+x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} 1+x_2 \\ y_2 \end{bmatrix}$ So it's not.

b) $T\begin{bmatrix} x_1+x_2 \\ y_1+y_2 \end{bmatrix} = \begin{bmatrix} y_1+y_2 \\ x_1+x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ x_1 \end{bmatrix} + \begin{bmatrix} y_2 \\ x_2 \end{bmatrix} = T\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + T\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$

$T\begin{bmatrix} cx_1 \\ cy_1 \end{bmatrix} = \begin{bmatrix} cy_1 \\ cx_1 \end{bmatrix} = c\begin{bmatrix} y_1 \\ x_1 \end{bmatrix} = c\,T\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ it's a linear transf.

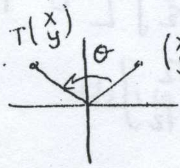
c) linear.

d) not linear.

(2) 

$T\binom{x}{y} = \binom{-x}{y}$ $\quad T\binom{1}{0} = \binom{-1}{0}$, $\quad T\binom{0}{1} = \binom{0}{1}$

$[T] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$.

(b) 

Rotation counterclockwise has the standard matrix.

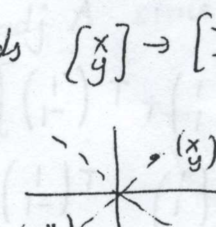
$[T] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ for $\theta = 45$.

$[T] = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$.

(3) $[T] = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

$[T]\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ -x \end{bmatrix}$ Hence T sends $\begin{bmatrix} x \\ y \end{bmatrix} \to \begin{bmatrix} -y \\ -x \end{bmatrix}$

which is a reflection in the line $y = -x$



(4) Let R: be the reflection in the line $y = x$

P: rotation around the origin by $60°$.

Then $T = P \circ R$ is the composition of two and recall their standard matrices are related by

$[T] = [P][R]$ $\qquad [R] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$[P] = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$

Hence $[T] = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -\sqrt{3}/2 & 1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$