Find a linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ s.t $T\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}2\\3\end{bmatrix}$ and $T\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}0\\5\end{bmatrix}$.

(8) Suppose that $T: \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation and suppose that v is a vector s.t $T(v) \neq 0$ but $T^2(v) = 0$ (where $T^2 = T \circ T$). Prove that v and $T(v)$ are linearly independent.

(9) Given the matrix $A = \begin{bmatrix}-1 & 3 & 0\\3 & -1 & 0\\-2 & -2 & 6\end{bmatrix}$ find a) char. poly b) the eigenvalues c) a basis for each eigenspace.

~~(10) Let $A = \begin{bmatrix}3 & 4 & 3\\5 & 7 & 2\\0 & 0 & 1\end{bmatrix}$ Find a) adj A b) A^{-1}~~

(11) Evaluate the following determinants

a) $\begin{vmatrix}0 & 1 & 2\\-1 & 0 & -3\\2 & 3 & 0\end{vmatrix}$ b) $\begin{vmatrix}a & b & c\\c & a & b\\b & c & a\end{vmatrix}$

(12) Let $A = \begin{bmatrix}4 & 1 & -1\\2 & 5 & -2\\1 & 1 & 2\end{bmatrix}$ Find a) eigenvalues of A
b) eigenvectors
c) A basis for the eigenspaces of A
d) Is A diagonalizable? If so, find an invertible matrix P s.t $D = P^{-1}AP$ is diagonal.

(13) Use the Cayley-Hamilton theorem (i.e every matrix satisfies its characteristic equation) to compute A^2, A^3.

(14) Let A be a 4x4 matrix with eigenvalues 5, 2, −2. If the eigenspace for $\lambda = 2$ is two dimensional, can we conclude that A is diagonalizable? Explain.

(15) Diagonalize the matrix $A = \begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}$

(16) True or false: Either give a proof or a counterexample.
a) If an nxn matrix A has 2 identical rows, then det A = 0
a) If an nxn matrix A and rank(A) = n then det A = 0.
b) If A is an nxn matrix and ... $Ax = 0$ for some $x \neq 0$