MATH 120R - LAB 7

Daylight and SAD

Over the course of a year, the length of the day - that is, the number of hours of daylight, calculated by subtracting the time of sunrise from the time of sunset- changes every day. Here is a table giving the length of day, rounded off to the nearest tenth of an hour, for Boston, latitude 42 N.

Date	Day	Daylight hrs	Date	Day	Daylight hrs
12/31	0	9.1	7/9	190	15.1
1/10	10	9.3	7/19	200	14.9
1/20	20	9.6	7/29	210	14.6
1/30	30	9.9	8/8	220	14.2
2/9	40	10.3	8/18	230	13.8
2/19	50	10.8	8/28	240	13.3
3/1	60	11.2	9/7	250	12.9
3/11	70	11.7	9/17	260	12.4
3/21	80	12.2	9/27	270	11.9
3/31	90	12.7	10/7	280	11.5
4/10	100	13.1	10/17	290	11.0
4/20	110	13.6	10/27	300	10.6
4/30	120	14.0	11/6	310	10.1
5/10	130	14.4	11/16	320	9.8
5/20	140	14.8	11/26	330	9.5
5/30	150	15.2	12/6	340	9.2
6/9	160	15.3	12/16	350	9.1
6/19	170	15.3	12/26	360	9.1
6/29	180	15.3			

Plot these points (day of year versus hours of daylight) on your graphing utility. The shape you should see should look like a rough approximation of a sine wave. In fact, the graph can be approximated by a function of the form

$$f(x) = A\sin(B(x + C)) + D.$$

Our challenge here is to determine values for the constants A, B, C and D.

First, we might look at A, the amplitude. We can find the amplitude of a sine function by taking half the difference between the highest point and the lowest point on the curve. Since we don't have data for every day of the year, and since the values have been rounded off, we don't know exactly what the highest and the lowest values are or on which days they occur. What wight be reasonable numbers to try for the highest and lowest values, given the data that we do have?

Next, let's try to determine B, which provides information on the period of the function. The formula for the period of any sine function is $P = \dfrac{2\pi}{B}$. use this to help you find B.

What does the constant C tell us? Remember our work with transformations of functions. The graph that you have of this data only looks like a sine function if you imagine that it was shifted to the left or to the right. Determine the point on the graph where it appears that a sine function does start, and use this to help you find C.

The constant D tells how far up or down from the $x-$ axis the graph is shifted. Compare this sine curve with a standard sine curve of the same amplitude and period. How far up has this one been shifted? Knowing the midline will help you find the value of D.

Try out your function now that you have values for the four constants; that is, plot $f(x) = Asin(B(x + C)) + D$ on the same coordinate system as your data. How well does it match the data? You may wish to adjust one of the constants to get better fit. Remember that the data is not a perfect sine wave, since the apparent path of the sun is actually one that is slightly elliptical rather than perfectly circular. This is a problem that we are not going to attempt to fix in this lab.

SAD (Seasonal Affective Disorder)

The seasons affect everyone's moods to some degree, but some people are so strongly affected by the amount of daylight that they experience severe depression during that part of the year when the hours of the daylight are shortest. Many magazine articles have been written about this condition, which has been termed seasonal affective disorder, or SAD. A typical person with SAD feels depressed for 2 or 3 months, sometime between the end of October and late February. She or he (women are affected more often than men) may experience a lack of energy and a craving for carbohydrates, and she/he may respond by oversleeping, overeating and withdrawing from society. An estimated 6% to 8% of the population of New England suffers from full-blown SAD.

Unlike the traditional treatments for other forms of depression, an effective therapy for SAD has the patient sit in front of bright lights every morning. If we assume that one hour of light therapy is equivalent to an hour of natural daylight, approximately how many hours of light therapy might a person with SAD require on a day in early January if she wanted to wake up for the "missing " hours of natural daylight (compared to March 21, the first day of spring)?

The graph represents results from a study of SAD patients and a group selected at random from the New York City telephone book, in which they were asked to specify the months in which they felt best or worst. Each point shows the proportion of people feeling at their best or worst in a particular month. "Feeling worst" is counted a a negative value.

If the seasonal mood fluctuations of SAD patients could be approximated by a sine function, would the function be in phase with the length-of-day function or out of phase? Explain.

SAD is often more prevalent further north. Here are some data for Fairbanks, Alaska, latitude 65 N. Plot these points and do the same thing that you did with the Boston data: find the constants A, B, C, and D so that the function $f(x) = A\sin(B(x + C)) + D$ models this data.

Note that the Fairbanks data is less precise, so the sine curve may seen to fit the data less accurately than the Boston data. Do not take any single point too seriously- simply look at the data as a whole.

Date	Daylight hrs	Day	Daylight hrs
0	3.7	190	20.6
10	3.9	200	20.3
20	4.2	210	18.8
30	5.7	220	18.4
40	8.2	230	15.9
50	8.6	240	15.5
60	9.1	250	15.0
70	10.7	260	13.4
80	12.2	270	11.0
90	13.7	280	10.5
100	13.7	290	8.9
110	15.7	300	8.4
120	16.2	310	8.0
130	18.6	320	5.6
140	20.2	330	4.0
150	20.5	340	3.8
160	20.6	350	3.7
170	20.7	360	3.7
180	20.7		

Plot the data for Boston and Fairbanks and their respective sine curves (the two cities' data should not be graphed on the same coordinate system; the data and the respective sine curves should be on the same coordinate system). Make sure to include the equations you found to model this data. Answer the few questions about SAD contained in the lab information.

The incidence and severity of seasonal affective disorder seem to depend on latitude. Scientific American (Jan 1989) reports that 24% of the population of Tromso, Norway (latitude 69 N), may suffer from midwinter insomnia, another manifestation of SAD. Tromso is so far north (more than 200 miles above the Arctic Circle) that the people there do not even see the sun between November 20 and January 20. They do, however, enjoy 24 hours a day of sun during the summer! Draw a sketch of what the lenght-of-day function for Tromso might look like. Is it a sine function? In the Southern hemisphere where the pattern of daylight is reversed, SAD reaches its peak during June and July. Draw the length-of -day graph for Wellington, New Zealand, or Puerto Montt, in south central Chile, both of which are approximately as far south of the equator as Boston is north of the equator (i.e. their latitude is 42 S). Give a formula that could represent that graph.

Using the length-of-day function you wrote for Boston, examine the graph and find the season of the year during which the days are lengthening most rapidly (how can you tell from the graph when this is happening?) Zoom in on that section, magnifying several times, until the portion you see resembles a straight line. Use the trace feature to find the coordinates of 2 points on that line and calculate its slope. Use that information to determine how rapidly the days are lengthening at that time of year. Now convert your calculation into minutes per day. How many minutes per week is this?

Repeat this same exercise for Fairbanks function. Compare your results with those from Boston. Approximately how many more times rapidly are the days changing in Fairbanks?