

MATH 120R - FINAL review

1. Given below are values from four functions: f, g, h and k . One function is periodic, one is a power function, one is exponential, and one is linear. (Note: some values on the table have been rounded to the nearest hundredth.)

x	1	2	3	4	5	6
f(x)	-1.34	-0.94	-0.54	-0.14	0.26	0.66
g(x)	64.0	16.0	4.00	1.00	0.25	0.06
h(x)	0.50	1.41	2.60	4.00	5.59	7.35
k(x)	0.25	0.5	0.75	0.50	0.25	0.50

- a) Tell which function is which, and give a reason for each answer:
- $f(x)$ is ... because ...
- $g(x)$ is ... because ...
- $h(x)$ is ... because ...
- $k(x)$ is ... because ...
- b) Find a formula for the linear function. Do that algebraically, and show all your work.
- c) Find a formula for the power function. Do that algebraically, and show all your work.
- d) Find a formula for the exponential function. Do that algebraically, and show all your work.
2. Let L be the line $2x + 3y = 7$.
- (a) Find the equation of the line perpendicular to L , that goes through the point $(1,2)$. Write your final answer in slope-intercept form.
- (b) Find the **exact coordinates** of the point of intersection of the line L with the line $4x - 11y = 7$.
3. Find the equations of the lines parallel to and perpendicular to the line $y + 4x = 7$, and through the point $(1,5)$.
4. A company believes that there is a linear relationship between the consumer demand for its products and the price charged. When the price was \$10 per unit, the quantity demanded was 520 units per week. When the unit price was raised to \$15 per unit, the quantity demanded was 434 units per week. Define $D(p)$ as the quantity per week demanded by customers at a unit price of p dollars. Assume that $D(p)$ is a linear function.
- (a) Find a formula for $D(P)$.
- (b) Give a practical interpretation of the slope of the line you found in part ((a))

- (c) Calculate $D(30)$ and $D^{-1}(30)$, and give practical interpretations of your answers.
5. $f(x) = 4x^2 - 4x - 8$
- Find the zeros of $f(x)$.
 - Complete the square to find the vertex of $f(x)$.
 - When is the function increasing, or decreasing? Specify x -intervals.
 - When is $f(x)$ concave up or down? Specify x -intervals.
 - Recall that all quadratic functions can be thought of simply transformations of the function $y = x^2$. List (in order) the transformations that need to be performed on the graph of $y = x^2$ to create the graph of f .
 - Describe the long run behavior of $g(x)$
6. The population of a town increases from 10000 to 50000 between 1960 and 1990. Assuming that the population grows exponentially, complete the following:
- Find a formula for the population of the town, P , s a function of time, t , measured in years after 1960.
 - What was the population of the town in 1970?
 - In what year did the population reach 30000?
 - Find the average rate of change of the population of the town, in thousands of people per year, between 1960 and 1970.
7. The radioactive element Strontium-90 has a half-life of 29 years. Suppose that a 15-gram sample of Strontium-90 is placed in a laboratoty.
- Find a formula for the amount, Q , of the sample that remains after t years.
 - How long will it take until only 3 grams of the sample remains? Round your final answer to the nearest 0.01 of a year.
 - What is the continuous decay rate of Strontium-90? Round your final answer to the nearest 0.01 of a percent.
8. The population of a bacteria colony at $t = 0$ is 500. After 2 hours, the population has grown to 730. Assuming that the population is growing exponentially, answer the following questions.
- Find a formula for the population, P , of the bacteria colony, as a function of time, t .
 - What is the doubling time of the population?
 - Find the average rate of change of the population of the bacteria colony between $t = 0$ and $t = 4$ hours.

9. Suppose that we have a function $f(x)$, where $f(3) = 10$ and $f(7) = 90$. Use this information to answer the following:
- (a) Define what it means for a function to be “linear”. Then, assuming $f(x)$ is a linear function, find a formula for $f(x)$
 - (b) Define what it means for a function to be “exponential”. Then assuming $f(x)$ is an exponential function, find a formula for $f(x)$. After this, clearly state what the “base” of your function is. Then explain what this base specifically tells you about the way the function is increasing or decreasing.
 - (c) Assuming f is a power function, find a formula for $f(x)$.
10. A parabola has x -intercept 3 and -1 and y -intercept 18. Find the equation for the parabola.
11. Solve the following equations:
- (a) $t^2 - t - 6 = 14$
 - (b) $1 - \frac{1}{1+x} = 3$
 - (c) $2 \sin x + 1 = 0$
 - (d) $\frac{0.5}{7-2x} = \frac{2}{30+4x}$
 - (e) $4(3)^x = 6(11)^x$
 - (g) $\sin x - \cos^2 x = -1$ for $0 \leq x \leq 2\pi$.
 - (h) $\cos(2t) = \frac{1}{2}$
 - (i) $\sin(2t) = \sin t$
 - (k) $\log x - \log(x-3) = 1$
12. A ferris wheel is 20 meters in diameter and boarded in the six o'clock position from a platform that is 4 meters above the ground. The wheel completes one full revolution every 2 minutes. At $t = 0$ you are in the twelve o'clock position.
13. Suppose that $\cos \theta = t$. Find:
- (a) $\sin \theta$
 - (b) $\tan \theta$
 - (c) $\csc \theta$
 - (d) $\sin(2\theta)$
 - (e) $\sin(\cos^{-1} x)$
 - (f) $\cos(4\theta)$

14. Let $f(x) = \frac{3}{x-4}$ and let $g(x) = \frac{3+4x}{x}$
- Compute and simplify $f(g(x))$
 - Compute and simplify $g(f(x))$
 - Are $f(x)$ and $g(x)$ inverse functions? Justify your answers with a complete sentence, referring to your calculations in parts (a), and (b).
15. The following questions refer to the rational function: $g(x) = \frac{3(x^2 - x - 2)}{x(x^2 + 2x - 8)}$
- Find the zeros of $g(x)$.
 - Find all vertical asymptotes of $g(x)$.
 - Describe the long run behavior of $g(x)$.
 - Based on your answers above, sketch an appropriate graph of $g(x)$ that illustrates all these features.
16. True or False?
- If the point $(0.5, -7)$ is on the graph of an EVEN function, then the point $(-0.5, -7)$ must also be on its graph.
 - The domain of $\frac{\ln(2x-1)}{\log(x^2-1)}$ is $x > \frac{1}{2}$
 - All rational functions have vertical asymptotes.
 - A rational function can never cross its asymptotes.
 - $\log \frac{y}{z} = \log y - \log z$
 - $2 \ln(P) - \ln(Q) + \ln(R) = \frac{P^2 R}{Q}$
 - If $\tan t = \frac{1}{\sqrt{3}}$ then $\sin t = 1$ and $\cos t = \sqrt{3}$.
17. Look at problems 2,3 page 221, 19 page 222, 22-25 page 223.
18. Look at problem 16 page 423, 19m page 424, 31 page 425.
19. Look at the tests, and the test reviews.