DETERMINANT PRACTICE

(1) Find the following determinants:

(a)
$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 1 & 2 \\ 2 & c \end{pmatrix}$$
(what is the condition on c so that this determinant is not 0).

(c)
$$\begin{pmatrix} 1 & 2 & 6 \\ 4 & -1 & 3 \\ 5 & 1 & 2 \end{pmatrix}$$

(d)
$$\begin{pmatrix} -5 & 2 & 12 \\ 9 & 9 & -9 \\ -1 & 13 & 15 \end{pmatrix}$$

(e)
$$\begin{pmatrix} a & b & 0 & c \\ d & e & 0 & f \\ 1 & 1 & 1 & 1 \\ g & h & 0 & i \end{pmatrix}$$

(f)
$$\begin{pmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

(g)
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & -1 \\ 0 & 0 & -1 & 3 & 6 \\ 0 & 0 & 2 & 1 & 7 \end{pmatrix}$$

(h)
$$\begin{pmatrix} 5 & 2 & -9 & 1 & 3 \\ 4 & -1 & 7 & 2 & 1 \\ 5 & 2 & -9 & 1 & 3 \\ 1 & 2 & 7 & -4 & -4 \\ 0 & 1 & 5 & 2 & 3 \end{pmatrix}$$

(i)
$$\begin{pmatrix} a & b & c & d \\ 2a-3e & 2b-3f & 2c-3g & 2d-3h \\ e & f & g & h \\ i & j & k & l \end{pmatrix}$$

(j)
$$\begin{pmatrix} 8 & 6 & 0 & 6 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

(k)
$$\begin{pmatrix} a & 0 & 0 & 0 \\ e & b & 0 & 0 \\ f & g & c & 0 \\ h & i & j & d \end{pmatrix}$$

(l)
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(m)
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(n)
$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(2) If A is an $n \times n$ matrx, and det(A) is c, what is det(kA), where k is any constant? You don't need a proof, but give a plausible explanation (not just an example).

(3) Is A is a singular (not invertible) $n \times n$ matrix, then prove (using determinants) that AB is also singular, where B is any $n \times n$ matrix.