Graphing Calculators 101

A graphing calculator can be a powerful tool for solving many of the problems we face in algebra – *if* you use it correctly. This guide is meant to give you an introduction on how to analyze the graph of a function using the TI-8X calculators. This is an easy way to calculate intercepts (zeros), maxima and minima, and intersections of graphs. A word of caution: **a calculator can only do the math that you tell it to do.** This means you still have to *know* the mathematics. It is **NOT** a replacement for the critical thinking process!

1 The viewing window

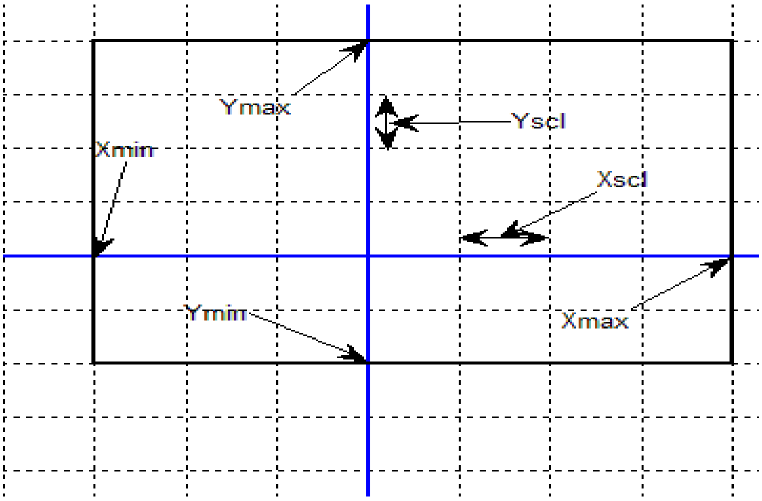
To be able to analyze a graph, you must be able to see the graph. The screen on a graphing calculator shows only a portion of the coordinate plane, so you may have to adjust the window in order to see a complete graph. The visible portion of the coordinate plane is determined by four numbers: *Xmin, Xmax, Ymin,* and *Ymax.* These define the left, right, bottom, and top boundaries of the viewing window. To change these values, press the WINDOW button. The distance between the marks on the axes is determined by *Xscl* and *Yscl.* If you are viewing a graph that requires large values for *Xmin, Xmax, Ymin,* or *Ymax*, make sure you adjust *Xscl* and *Yscl* accordingly, otherwise the axes could become cluttered.

If, during the process of adjusting the window, you need to start over, you can always get back to default window by pressing the ZOOM button, and selecting ZSTANDARD. The default window is $[-10, 10] \times [-10, 10]$.

1.1 Finding a good viewing window

A complete graph of a function should include all of its important features: intercepts, turning points, and end behavior. Ideally, you should have some rough idea of what to expect the graph to look like *before* you plug your function into the $y=$ menu. For example, some things you can easily calculate for determining a window are the y-intercept, number of zeros, and location of asymptotes.

However, sometimes you may have no idea where the important features are. The ZOOM menu gives us some options for quickly changing the viewing window, either to get an idea of where the graph is or to see more detail in a particular region.



The viewing window is determined by *Xmin, Xmax, Ymin,* and *Ymax*. The spacing of the tick marks is determined by *Xscl* and *Yscl*.

One way to find a good window: use Zoom Out (in the ZOOM menu) until you can see the complete graph, then adjust the window parameters by hand (using the WINDOW button) to eliminate wasted space. An alternative to adjusting the parameters by hand is to use ZBox (also in the ZOOM menu) to draw a box around the important features using the cursor.

Finding the appropriate window is often a trial-and-error process, and is usually the most difficult part of analyzing a graph with the calculator. However, it is an essential first step. A fully-worked example is given in Section 4, and some suggested problems are given in Section 5. As with most things, practice is the only thing that will make this process easier.

Once you have an appropriate window, you can calculate precisely the zeros, turning points, and intersections of graphs.

2 The CALC menu

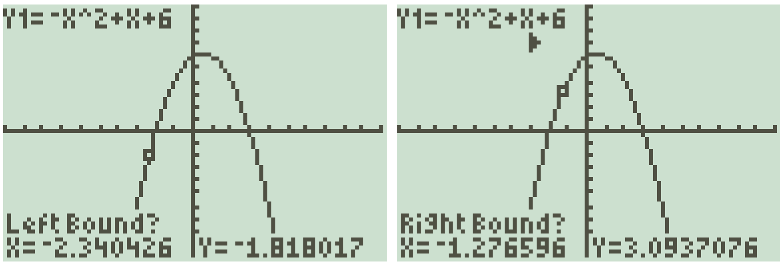
This is the menu that holds most of the functions that we will use. To unleash its magic, press 2ND–TRACE.

2.1 value

This feature lets you evaluate the function for any x-value that you wish. Simply type the number and press ENTER, and the calculator will spit out the y-value. If you need to do this for many different x-values, using the TABLE feature may be faster; press 2ND–GRAPH to reach the TABLE.

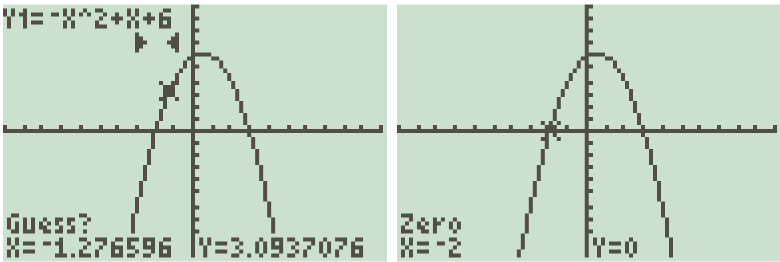
2.2 zero (root)

This is the feature that calculates the x-intercepts of the graph. In order for the calculator to do this, you must tell it where to look. It will ask you for a LEFT BOUND, a RIGHT BOUND, and a GUESS. The LEFT (or LOWER, on the older models) BOUND is meant to be an x-value that is to the *left* of the desired intercept. Move the cursor using the arrow keys until it is situated on a point to the left of the intercept – note that this could be above or below the x-axis, depending on what the graph is doing – and press enter.



Choose a LEFT and a RIGHT BOUND by moving the cursor to a point that is to the *left* or *right* of the intercept you are trying to calculate.

It will then ask for the RIGHT (or UPPER) BOUND, so move the cursor to a point that is to the *right* of the intercept and press ENTER. The two points you just chose should be on opposite sides of the x-axis. After you choose the left and right bounds, look for arrows that appear in the window: they should be pointing towards each other, with the intercept in between.



Make sure the arrows are pointing towards each other when it asks you to guess.

Next it will ask you to GUESS, so place the cursor as near as you can to the intercept and press ENTER one more time. It will spit out the location of the x-intercept, as well as the corresponding y-value.

A note about accuracy: If you get something like 7.2E-11 for the y-value, don't worry. This is shorthand for $7.2 \times 10^{-11} = 0.000000000072$, which is practically zero. Just remember: everything the calculator does is an approximation only good to 10 significant digits. For an example of why you should not blindly trust your calculator, and why it's extremely important to still use critical thinking at every step, perform this calculation:

$(10^{15} + 7.2 - 10^{15}) * 100$. What does your calculator tell you the answer is? What is the true answer? What's going on?

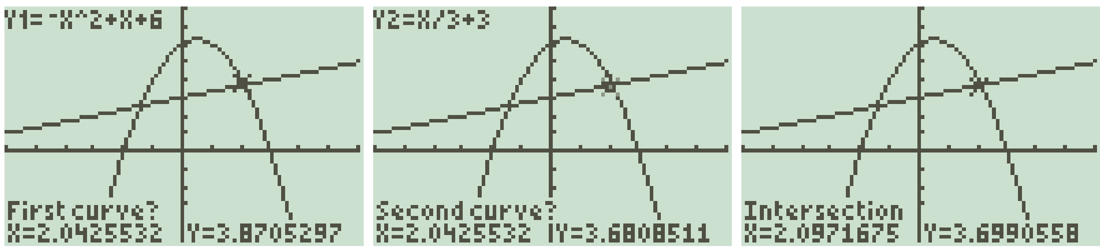
Remember that you have to be able to *see* the x-intercept on the graph before you can choose your left and right bounds. Sometimes you may have to zoom in on a particular zero to get a better result.

2.3 minimum and maximum

These features are what you use to calculate turning points. Again, you must tell the calculator where to look by giving it a LEFT (LOWER) and a RIGHT (UPPER) BOUND. The process works exactly the same as finding zeros. Choose a LEFT BOUND that is to the left of the turning point, then a RIGHT BOUND. Make sure the arrows are pointing towards each other, with the max/min in between. Place the cursor as near as you can to the actual max/min for the GUESS, and press enter. The calculator will spit out both the location (x-value) of the turning point, as well as the maximum or minimum value (y-value). See the example in Section 4.

2.4 intersect

This feature allows you to calculate the intersection of two different graphs. To use it, you must be able to clearly see the intersection point in the viewing window. The calculator will ask you to identify the FIRST CURVE. It is very important that you move the cursor *as close as you can* to the intersection point before pressing ENTER, in case there is more than one intersection.



To calculate the intersection of two graphs, use the INTERSECT feature under the CALC menu. Place the cursor as near as you can to the point of intersection and press ENTER three times.

Press ENTER again to select the SECOND CURVE, and press ENTER again to give it a GUESS (you usually don't need to readjust the cursor). It will spit out the x- and y-values of the intersection.

3 Common error messages

- **ERR: SYNTAX** The calculator does not like what you typed into $y =$ menu. Check it carefully. Often, the mistake is that you use – instead of (-) for a negative sign. You may also get this error if your parentheses are horribly mismatched. Also check your WINDOW settings for the (-) sign.

- **ERR: WINDOW RANGE** This probably means you entered inconsistent values for *Xmax, Xmin, Ymax,* or *Ymin.* Check that $Xmin < Xmax$ and $Ymin < Ymax$.

- **ERR: NO SIGN CHNG** This error arises when calculating x-intercepts. When choosing your LEFT and RIGHT BOUND, make sure that the two points lie on opposite sides of x-axis. One should lie above (with positive y-value), and the other should lie below (negative y-value).

- **ERR: BOUND** This can arise when calculating zeros or maxima/minima. When you choose your LEFT and RIGHT BOUND, always make sure that the arrows are pointing *towards* each other. In other words, you may have reversed what was LEFT and what was RIGHT.

4 Example

Consider the graph of

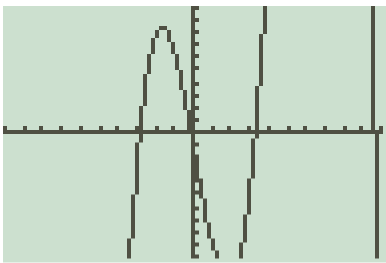
$$y = x^3 - 8x - 2^x.$$

1. Determing the largest x-intercept.

2. Find the maximum value of y and where this maximum occurs.

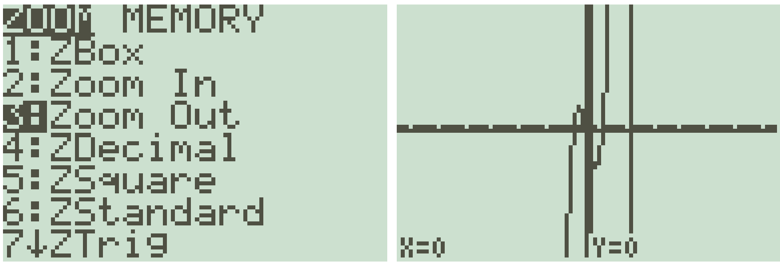
3. Solve the equation when $y = 80$.

Step 1: Find a good window. After entering the equation into the $y =$ menu and pressing GRAPH, you will probably see something like this (make sure the window starts in the standard form):

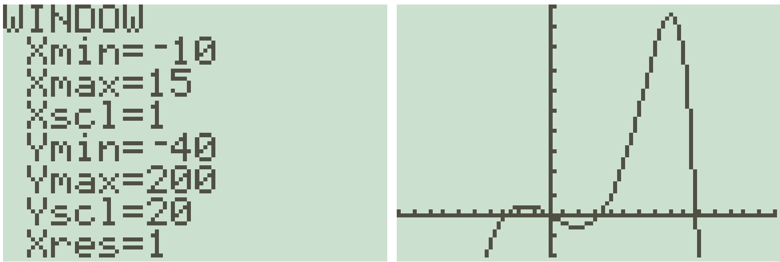


The maximum is not visible, zoom out!

To see the complete graph, you need to zoom out. After zooming out, you can adjust the window manually. It looks like the horizontal extent of the graph is relatively small, but the vertical range still needs to be extended. We can shrink the horizontal range by making *Xmin* larger (closer to zero) and *Xmax* smaller. Similarly, we can extend the height of the graph by making *Ymax* bigger. Play around with these numbers until you have a graph that looks nice.



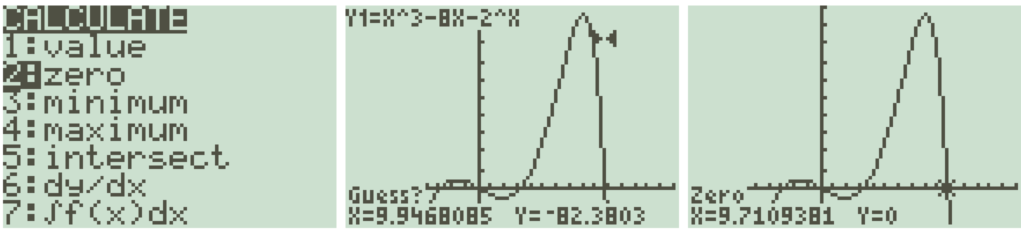
After zooming out, things get cluttered.



Adjust the window settings manually to fine-tune the picture. This will probably take some trial-and-error.

Now you have a complete picture of the graph, and you can begin to calculate intercepts and maxima.

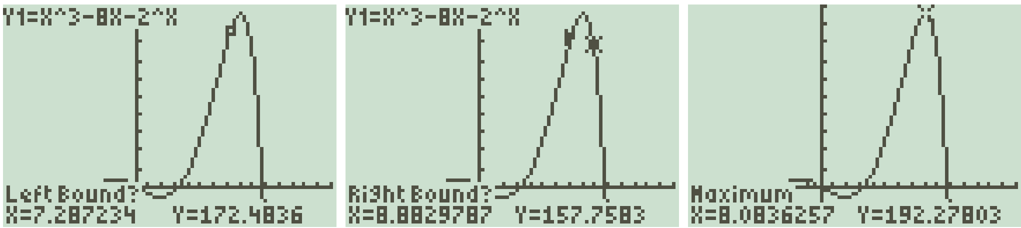
Step 2: Calculate the largest zero. You are interested in the x-intercept farthest to the right. In the CALC menu, select ZERO. When it asks for a LEFT BOUND, move the cursor to a point that is to the left of the intercept. Do the same for the RIGHT BOUND, and then enter the GUESS. You should find that the largest x-intercept is at



Make sure the arrows are pointing towards each other before you guess!

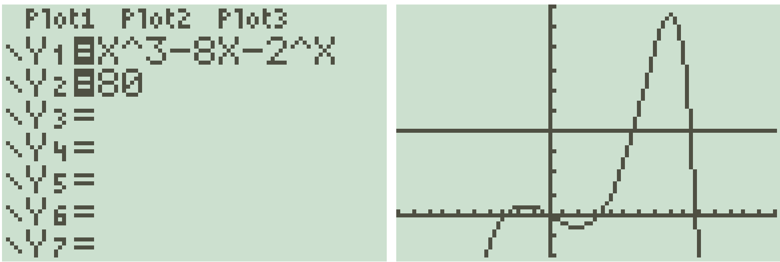
$x = 9.7109381$.

Step 3: Calculate the maximum. In the CALC menu, select MAXIMUM. Choose your LEFT and RIGHT BOUNDS to the left and right of the maximum, then give it a GUESS. You should find the maximum value at about $(8.084, 192.28)$.



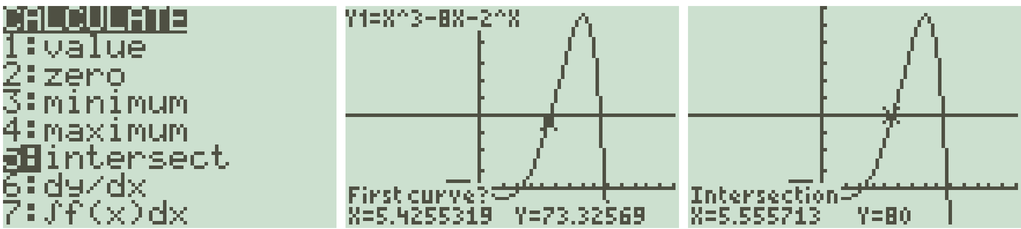
Again, make sure the arrows are pointing towards each other before you guess!

Step 4: Solving the equation for given y value Here you are asked to determine the x-value(s) when $y = 80$. Begin by drawing the line $y = 80$ in the $y =$ menu. Now that you



Again, make sure the arrows are pointing towards each other before you guess!

have two graphs, you can calculate the *intersection* of the two graphs, and the x-values of the intersection will give you the solution. Select INTERSECT from the CALC menu, and move the cursor as close as you can to the desired point of intersection (there are two in this problem, so we must do this twice), and press ENTER three times. You will



The x-value of the intersection gives the solution to $80 = x^3 - 8x - 2^x$.

find one solution to be $x = 5.555713$, and the other to be $x = 9.3990333$.

As you may be able to tell from this example, the hardest part of the whole process is finding the viewing window. Once you have that, the rest is just a matter of picking LEFT and RIGHT BOUNDS correctly, and the calculator does the rest. However, finding a good viewing window may take some practice.

5 Suggested problems

For these problems, find a complete graph of each function, and calculate any x-intercepts.

1. $y = x^3 - 4x - 16$

2. $y = x - 4^x + 24$

3. $y = |x - 15| - |x + 15| + 20$ (the absolute value function is found by pressing MATH, then RIGHT)

4. $y = \frac{30+3x^2}{x^2+5}$ (make sure to use parentheses!)

5. $y = x|x - 21|$

6. The equation $x = y^2 - 4$ cannot be graphed directly on a calculator (why?). However, we can solve for y to obtain $y = \pm\sqrt{x+4}$. To see the graph, we can plot $y = \sqrt{x-4}$ and $y = -\sqrt{x-4}$ simultaneously. Use this technique to graph the solutions of $x = y^2 - 4$ on the calculator.

7. Use the technique outlined in #6 to graph the equation $x^2 + (y-2)^2 = 16$. What should the shape of this graph be? Does your calculator screen agree with your intuition? This is an example of why it's important to know something about your equations *before* you enter them in the calculator.