Exam 4 review problems

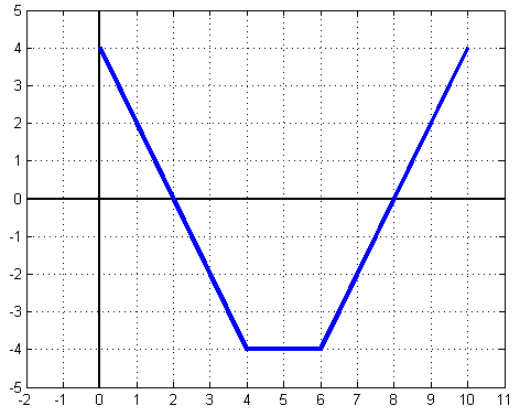
1. A bicyclist named Jehoshaphat is riding along a very hilly road, so that his speed increases and decreases. His velocity v in ft/sec is recorded in the table below in 4 second intervals during a portion of his trip.

t	0	4	8	12	16	20
v	10	12	9	12	25	22

   (a) Find left- and right-hand Riemann sums to approximate the distance that Jehoshaphat travels during this 20 second interval.

   (b) Find upper and lower bounds for the distance that Jehoshaphat travels.

2. The figure below shows a graph of $y = f(x)$.



   (a) Find $\int_0^7 f(x)dx$.

   (b) If $F(10) = -10$, where $F'(x) = f(x)$, find $F(8)$.

3. (a) The number of workers w used during an environmental cleanup project is a function of the number of hours since the project began. What are the units of $\int_0^{24} w(t)dt$?

   (b) The current in a wire is defined as the time derivative of the charge: $I(t) = \frac{dQ}{dt}$. What does $\int_a^b I(t)dt$ represent?

4. A weed growing in a field spreads at a rate $R(t) = 5\ln(t+1)$, where R is measured in square meters per day and t is measured in days. If the weed initially covers 100 square meters, write an expression to give the total amount of area infested by weed after 7 days. Use your calculator to evaluate the expression to one decimal (or try to do it by hand!).

5. Suppose $f(t)$ is an even function and $g(t)$ is an odd function, and that $\int_0^{10} f(t)dt = 12$ and $\int_0^{10} g(t)dt = -3$. Evaluate the following definite integrals.

(a) $\int_0^{10} \big(3f(t) - 4g(t) + 5\big)dt$

(b) $\int_{10}^{0} \left(-\frac{g(t)}{6}\right) dt$

(c) $\int_{-10}^{0} g(t)dt$

(d) $\int_{-10}^{10} \left(f(t) + g(t)\right) dt$

6. Evaluate the following indefinite integrals.

(a) $\int \left(a^2 x + \sqrt{b}\right) dx$

(b) $\int \left(e^{2t} + \frac{1}{2t}\right) dt$

(c) $\int z^2 \left(z^4 + 2\right)^2 dz$

(d) $\int \frac{3}{\sqrt{kt}} dt$

7. Evaluate the following definite integrals.

(a) $\int_{-5}^{5} \frac{e^x}{2} dx$

(b) $\int_{1}^{5} \frac{C}{t} dt$

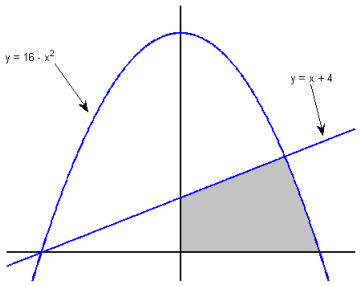
(c) $\int_{0}^{1} x^{n-1} dx$

(d) $\int_{1}^{8} \frac{8}{3\sqrt[3]{x^5}} dx$

8. Find the antiderivative $H(s)$ of $h(s) = 3(s+1)^2$ that satisfies $H(1) = 4$.

9. (a) Use your calculator to find the area between the curve $y = (x-1)^3 - 9x + 9$ and the x-axis. Round to one decimal.

   (b) Find the exact area of the shaded region below. It is bounded by the curves $y = 16 - x^2$, $y = x + 4$, and the x and y axes.



10. Let $F(x)$ be the curve defined by $F(x) = \int_0^x \frac{1}{1+t+t^2} dt$.

   (a) Evaluate $F(0)$

   (b) Find $F'(x)$

   (c) Find the intervals where F is increasing and decreasing.

   (d) Find the intervals where F is concave up and concave down.