Exam 2 review problems

1. Find and classify the critical points of the following functions
   (a) $f(x) = \sqrt[3]{x^2 - 1}$
   (b) $g(x) = \frac{1}{6}x^3 + |x - 1|$
   (c) $A(h) = \frac{h}{2}\sqrt{(L-h)^2 - h^2}$

2. (a) A function has first derivative given by $f'(x) = \frac{x^2(x-2)^3(x+1)}{\sqrt{x+2}}$. Find and classify the critical points of $f(x)$.
   (b) A function has second derivative given by

$$g''(t) = -e^{-t}(\ln(t) - 1)^2(t^2 - 1)^3(1 - \cos(x))$$

   . Determine where $g(t)$ has inflection points.

3. State whether the following functions have a global maximum or a minimum on the given interval. If global extrema exist, find them; otherwise, provide an accurate upper and lower bound.
   (a) $f(x) = \frac{1}{x^2}$ on the interval [-3,2].
   (b) $f(x) = x + 2\cos(x)$ on the interval $[-\pi, \pi]$.
   (c) $\theta(t) = t\sqrt{-t^2 - t + 3}$ on (-2,0).

4. (a) Find all critical points of $f(x) = ax^2e^{-bx}$, where a and b are constants.
   (b) Find the values of a and b so that f has a critical point at the point (5,12).
   (c) Identify each critical point as a local maximum or local minimum of f in part (b).

5. The surface area of a rectangular box with length x, width y, and height z, all in meters, is changing at a rate of 5 m^2/min. At a point in time when the box has dimensions $x = 4$, $y = 3$, and $z = 1$, find how fast the height is changing if the length is increasing at 2 m/sec and the width is decreasing at 1.5 m/sec.

6. Determine if the following limits can be calculated using l'Hopital's rule. Calculate the limit.

   (a) $\lim_{x\to 0} \frac{x}{\arctan(x)}$

   (b) $\lim_{x\to \pi} \frac{\sin(x)}{x}$

   (c) $\lim_{x\to 0^+} \big(-\ln(x)\big)^x$

   (d) $\lim_{x\to \infty} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$

   (e) $\lim_{x\to \infty} e^{-x}3^x$

   (f) $\lim_{x\to 0} (3x+4)^{1/x}$

7. What is the largest possible area that a rectangle can have, if the sum of the diagonal and the length is always equal to 10? Provide an exact answer and prove that it is optimal.

8. A steel pipe is being carried down a hallway 10 feet wide. At the end of the hall there is a right-angled turn into a narrower hallway 8 feet wide. What is the length of the longest pipe that can be carried horizontally around the corner? Provide an exact answer and prove that it is optimal.