Exam 1 Review Problems — Solutions

1. a) $f(2) = -27$; b) $f(-2) = -27 \Rightarrow \dfrac{f(2)-f(-2)}{2-(-2)} = \boxed{0}$

b) $f(\pi/3) = \cos\left(\frac{2\pi}{3}\right) + \frac{\pi}{3} = -\frac{1}{2} + \frac{\pi}{3}$; $f\left(\frac{7\pi}{6}\right) = \cos\left(\frac{7\pi}{3}\right) + \frac{7\pi}{6} = \frac{1}{2} + \frac{7\pi}{6}$

$f\left(\frac{7\pi}{6}\right) - f\left(\frac{\pi}{3}\right) = \left(\frac{1}{2} + \frac{7\pi}{6}\right) - \left(-\frac{1}{2} + \frac{\pi}{3}\right) = 1 + \frac{5\pi}{6}$

$\frac{7\pi}{6} - \frac{\pi}{3} = \frac{5\pi}{6} \Rightarrow \dfrac{f\left(\frac{7\pi}{6}\right) - f\left(\frac{\pi}{3}\right)}{\frac{7\pi}{6} - \frac{\pi}{3}} = \dfrac{1 + \frac{5\pi}{6}}{\frac{5\pi}{6}} = \boxed{\dfrac{6+5\pi}{5\pi}}$

2. a) $q(x)$ decreases $\Rightarrow \boxed{q'(x) < 0}$

b) i. $q_L'(2) \approx \dfrac{2.65 - 4.90}{2-1} = \boxed{-2.25}$

ii. $q_R'(2) = \dfrac{0 - 2.65}{3-2} = \boxed{-2.65}$

iii. $q'(2) \simeq \dfrac{0 - 4.90}{3-1} = \boxed{-2.45}$ ← same as $\frac{1}{2}\left(q_L'(2) + q_R'(2)\right)$

c) i. $q_L'(-1) = \dfrac{8.16 - 10}{-1-(-4)} = \boxed{-0.613}$

Why?

ii. $q_R'(-1) = \dfrac{4.90 - 8.16}{1-(-1)} = \boxed{-1.63}$

iii. $q'(-1) \simeq \dfrac{4.90 - 10}{1-(-4)} = \boxed{-1.02}$ ← not the same as $\frac{1}{2}\left(q_L'(-1) + q_R'(-1)\right)$

d)

x	-4	-2.5	-1	0	1	1.5	2	2.5	3
$q(x)$	10		8.16		4.90		2.65		0
$q'(x)$		-0.61		-1.6		-2.3		-2.7	
$q''(x)$			-0.40		-0.47		-0.40		

← $\boxed{q''(x) < 0}$

e) $\boxed{q''(2) \simeq -0.4}$ (From table)

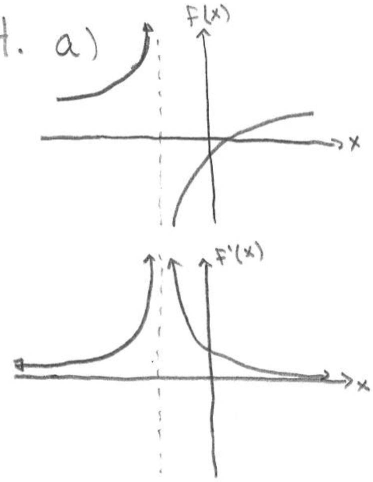
3. a) $f'(2) = \lim\limits_{h\to 0} \dfrac{\frac{3}{2(2+h)^2} - \frac{3}{2(2)^2}}{h} = \frac{3}{2} \cdot \lim\limits_{h\to 0} \dfrac{\frac{1}{(2+h)^2} - \frac{1}{4}}{h} = \frac{3}{2} \cdot \lim\limits_{h\to 0} \frac{1}{h}\left(\dfrac{4-(2+h)^2}{4(2+h)^2}\right)$

$= \frac{3}{2} \cdot \lim\limits_{h\to 0} \frac{1}{h}\left(\dfrac{-4h-h^2}{4(2+h)^2}\right) = -\frac{3}{2} \cdot \lim\limits_{h\to 0} \dfrac{4+h}{4(2+h)^2} = -\frac{3}{2} \cdot \frac{1}{4} = \boxed{-\frac{3}{8}}$

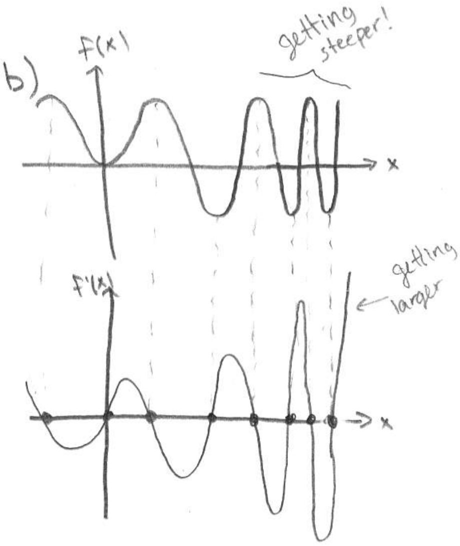
b) $f'(1) = \lim_{h\to 0} \dfrac{\sqrt{2(1+h)} - \sqrt{2}}{h} = \sqrt{2}\cdot \lim_{h\to 0} \dfrac{\sqrt{1+h} - 1}{h} \cdot \dfrac{-\sqrt{1+h} - 1}{-\sqrt{1+h} - 1} = \sqrt{2}\cdot \lim_{h\to 0} \dfrac{-h}{-h(\sqrt{1+h} + 1)}$

$= \sqrt{2} \lim_{h\to 0} \dfrac{1}{\sqrt{1+h} + 1} = \sqrt{2}\cdot \dfrac{1}{2} = \boxed{\dfrac{\sqrt{2}}{2}}$

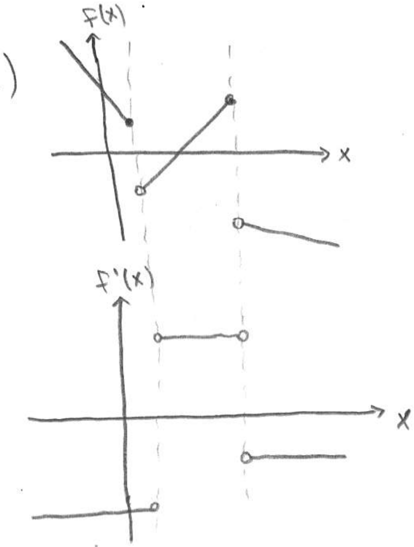
4. a)



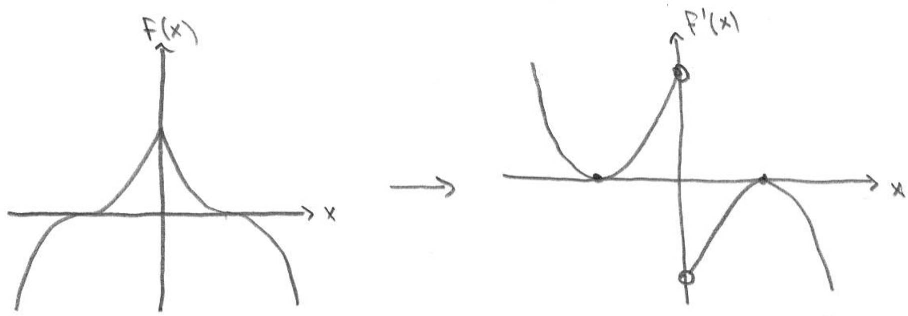
b)



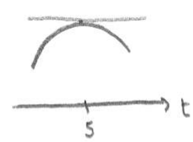
c)



d)



5. a) $r'(5) = 0$ and $r''(5) < 0 \Rightarrow$



$\Rightarrow$ The rate of snowfall is at its max 5 hours after the storm began.

b) $\dfrac{dr}{dt}$ has units of millimeters per minute per hour

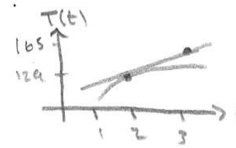
c) $r'(1) = 2.5$ means: One hour after the storm began, the rate of snowfall is increasing at a rate of 2.5 mm/min per hour.

OR: The rate of snowfall will increase by about 2.5 mm/min between the first and second hour of the storm.

6. a) $T(2) = 129$, $T\left(2+\frac{1}{60}\right) = 129.6 \Rightarrow T'(2) \approx \dfrac{129.6-129}{1/60} = \boxed{36\text{°F per hour.}}$

b) Tangent line: $y = 36(t-2)+129 \Rightarrow T(2.5) \approx 36(0.5)+129 = \boxed{147\text{°F}}$

c) $T''<0 \Rightarrow$ tangent line is OVER approximation



Use tangent line to estimate $T(3)$: $T(3) \approx 36(1)+129 = 165$°F

$\Rightarrow T(3) < 165$°F ! $\Rightarrow$ $\boxed{\text{Yes,}}$ you can safely ignore the turkey for an hour to watch football, BUT: you better get cracking on the sides + dessert instead!

7. a) $\dfrac{d}{dx}\left(2x^2 - 3e^{-3x}\right) = \boxed{4x+9e^{-3x}}$

b) $f(x) = \frac{a}{c}x^{-2} + \frac{b}{c}x^{-1} \Rightarrow f'(x) = -\frac{2a}{c}x^{-3} - \frac{b}{c}x^{-2} = \boxed{-\dfrac{2a}{cx^3} - \dfrac{b}{cx^2}}$

c) $H = z^{3/2} + 5z^2 \Rightarrow \dfrac{dH}{dz} = \frac{3}{2}z^{1/2} + 10z \Rightarrow \boxed{\dfrac{d^2H}{dz^2} = \frac{3}{4}z^{-1/2} + 10}$

d) $y = \pi x^{\log(2)} - \pi e^{\sqrt{2}} \cdot x^{\log(2)+\log(5)} + 2\pi x^{\log(2)+2.1}$ (Note: $\log(2)+\log(5)=1$)

$\boxed{\dfrac{dy}{dx} = \pi\log(2)\,x^{\log(2)-1} - \pi e^{\sqrt{2}} + 2\pi(\log(2)+2.1)\,x^{\log(2)+1.1}}$ ← Fugly!

e) $C = 2\left(\frac{1}{9}\right)^h - 8h^3 \Rightarrow \boxed{\dfrac{dC}{dh} = 2\ln\left(\frac{1}{9}\right)\left(\frac{1}{9}\right)^h - 24h^2}$

f) $A = P\left[\left(1+\frac{r}{n}\right)^n\right]^t \Rightarrow \dfrac{dA}{dt} = P\ln\left(\left(1+\frac{r}{n}\right)^n\right)\left[\left(1+\frac{r}{n}\right)^n\right]^t = \boxed{nP\ln\left(1+\frac{r}{n}\right)\left(1+\frac{r}{n}\right)^{nt}}$

8. a) $d(t) = 10t - t^2 \Rightarrow \boxed{d'(t) = 10-2t}$. The units are $\boxed{\text{cm/sec}}$

b) $d(t) = 9 \Rightarrow 9 = 10t - t^2 \Rightarrow t^2 - 10t + 9 = 0 \Rightarrow (t-9)(t-1) = 0 \Rightarrow t = 9, 1$.

$\boxed{d'(1) = 8 \text{ cm/s}}$ ← walking <u>up</u> the stick

$\boxed{d'(9) = -8 \text{ cm/s}}$ ← walking <u>down</u> the stick

c) $d''(t) = -2 \Rightarrow \boxed{d''(3) = -2 \text{ cm/s}^2}$

9. a) C, G, I

   b) E only

   c) Increasing: A, B, C, D, H, I, J ; decreasing: F, G

   d) Greatest: J, least: A

10. a) Increasing: A, E-J ; decreasing: C

    b) Local max at B; local min at D

    c) Global max at J

    d) Concave up: D, E, F, J; Concave down: A, B, H

    e) C, G, I (compare w/ 9a)

    f) Greatest: J, least: A (compare w/ 9d)

    g) Positive: A, B, C, D, H, I, J; negative: F, G (Compare w/ 9c)

11. For continuity, need $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) \Rightarrow m+b=6$

    For differentiability, need $\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^+} f'(x) \Rightarrow \frac{d}{dx}(mx+b)\Big|_{x=1} = \frac{d}{dx}(x^2+2x+3)\Big|_{x=1}$

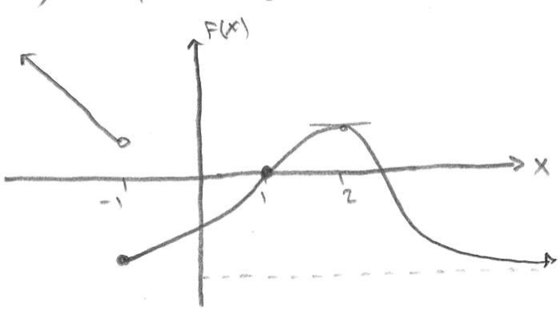
    $\Rightarrow \boxed{m=4}$ $\quad m+b=6 \Rightarrow \boxed{b=2}$

12. a) $\frac{d}{dx}(4-x^2)\Big|_{x=1} = -2x\Big|_{x=1} = \boxed{-2}$

    b) $\frac{d}{dx}(2e^{1-x})\Big|_{x=1} = -2e^{1-x}\Big|_{x=1} = \boxed{-2}$

    c) $\boxed{\text{No!}}$ Even though $\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^+} f'(x)$, $\boxed{f(x) \text{ is NOT CONTINUOUS at } x=1}$
    therefore NOT differentiable

13. a) One possible graph:



    b) One possible graph: