Exam 1 review problems

1. Find the average rate of change of $f(x)$ between $x = a$ and $x = b$

   (a) $f(x) = (1 - x^2)^3$, $a = -2$, $b = 2$

   (b) $f(x) = \cos(2x) + x$, $a = \pi/3$, $b = 7\pi/6$

2. The table below contains values taken from a function $q(x)$.

x	-4	-1	1	2	3
$q(x)$	10	8.16	4.90	2.65	0

   (a) Is $q'(x)$ positive or negative?

   (b) Estimate $q'(2)$

      i. using the point to the left;

      ii. using the point to the right;

      iii. using the point to the left and right. Is this the same as averaging your answers to (i) and (ii)?

   (c) Estimate $q'(-1)$

      i. using the point to the left;

      ii. using the point to the right;

      iii. using the point to the left and right. Is this the same as averaging your answers to (i) and (ii)?

   (d) Is $q''(x)$ positive or negative?

   (e) Estimate $q''(2)$.

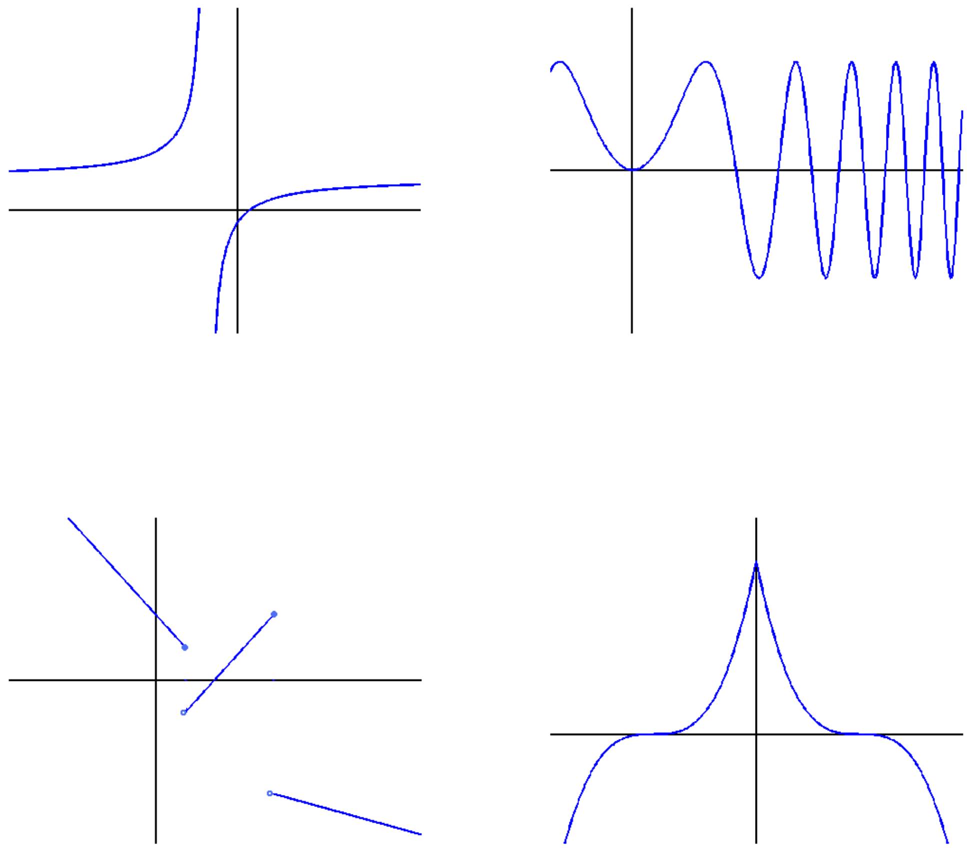
3. Using proper notation, calculate $f'(a)$ from its definition for the following functions

   (a) $f(x) = \dfrac{3}{2x^2}$, $a = 2$

   (b) $f(x) = \sqrt{2x}$, $a = 1$

4. The graphs below show a function $y = f(x)$. Sketch a graph of $y = f'(x)$.
Remember:

- Turning points/horizontal tangents of f should appear as zeros of f'
- Inflection points of f should appear as local maxes/mins of f'
- Points where f is non-differentiable should appear as discontinuities of f'



5. Let $r(t)$ represent the rate at which snow is falling during a winter storm (in somewhere that is *not* Tucson). The units of r are millimeters per minute, and t is measured in hours since the storm began.

(a) If $r'(5) = 0$ and $r''(5) < 0$, what can you say about the storm 5 hours after it started?

(b) What are the units of $\dfrac{dr}{dt}$?

(c) Interpret the statement $r'(1) = 2.5$. Write a complete sentence, include units, and be specific about the context of the problem.

6. Suppose you're having your parents over for Thanksgiving this year for the first time ever, and it is your job to not ruin the turkey. After two hours of cooking, you check the meat thermometer and it reads 129°F. Being paranoid about burning the turkey, you measure the temperature again after one minute, and find that it reads 129.6°F. Let $T(t)$ be the temperature of the turkey, in degrees °F, after t hours of cooking.

   (a) Estimate $T'(2)$. Be careful with units.

   (b) Estimate the temperature of the turkey after 2.5 hours of cooking.

   (c) Suppose $T''(t) < 0$, and the turkey needs to reach a temperature of 165°F. Meanwhile, the Seahawks and 49ers are locked in an epic struggle for Thanksgiving NFL dominance, and there is still one hour of nonstop gridiron action remaining in the game. After you have measured the temperature at the 2 hour mark, can you safely ignore the turkey for the next hour and watch football? Support your answer (a successful Thanksgiving depends on it). No, you cannot cook the turkey and watch football simultaneously.

7. Find the indicated derivative. Use proper notation.

   (a) $\frac{dy}{dx}$ for $y = 2x^2 - 3e^{-3x}$

   (b) $f'(x)$ for $f(x) = \frac{a + bx}{cx^2}$

   (c) $\frac{d^2H}{dz^2}$ for $H = \sqrt{z^3} + 5z^2$

   (d) $\frac{dy}{dx}$ for $y = \pi x^{\log(2)}(1 - e^{\sqrt{2}}x^{\log(5)} + 2x^{2.1})$

   (e) $\frac{dC}{dh}$ for $C = 2 \cdot 3^{-2h} + (-2h)^3$

   (f) $\frac{dA}{dt}$ for $A = P\left(1 + \frac{r}{n}\right)^{nt}$

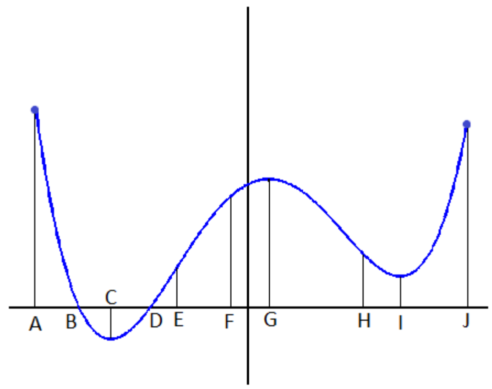
8. An ant named Anthony decides to go for a walk along the edge of a very thin branch. After t seconds of walking, his distance d, in centimeters, from the bottom of the branch is given by $d(t) = t(10 - t)$.

   (a) Find Anthony's speed after t seconds of walking. What are the units?

   (b) How fast is Anthony walking when he is exactly 9 centimeters from the bottom of the stick? Interpret the sign of your result(s).

   (c) Find Anthony's acceleration after 3 seconds of walking. What are the units?

9. The figure below shows the graph of a function $y = f(x)$. The questions below refer to the x-values label A-J.



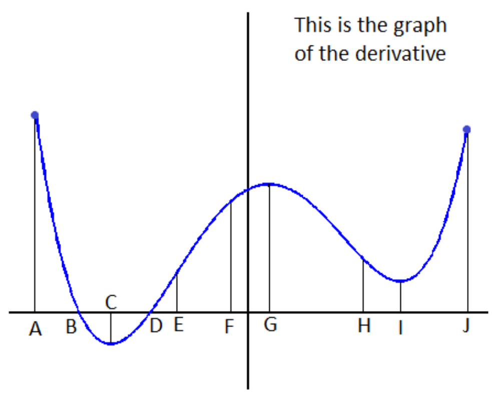
(a) At which point(s) is $f'(x) = 0$?

(b) At which point(s) is $f''(x) = 0$?

(c) At which points is $f'(x)$ increasing? Decreasing?

(d) At which point(s) is $f'(x)$ greatest? Least?

10. The figure below shows the graph of the derivative $y = f'(x)$ for a function f. The questions below refer to the x-values label A-J.



(a) At which point(s) is $f(x)$ increasing? Decreasing?

(b) At which point(s) does $f(x)$ have a local maximum? A local minimum?

(c) At which point does $f(x)$ have a global maximum?

(d) At which point(s) is $f(x)$ concave up? Concave down?

(e) At which point(s) does $f(x)$ have an inflection point?

(f) At which point is $f''(x)$ greatest? Least?

(g) At which point(s) is $f'''(x)$ positive? Negative?

11. Find values of m and b so that the function

$$f(x) = \begin{cases} mx + b & x \leq 1 \\ x^2 + 2x + 3 & x > 1 \end{cases}$$

is continuous and differentiable.

12. (a) Find $\left.\frac{d}{dx}\left(4 - x^2\right)\right|_{x=1}$

(b) Find $\left.\frac{d}{dx}\left(2e^{1-x}\right)\right|_{x=1}$

(c) Now consider the piecewise function

$$f(x) = \begin{cases} 4 - x^2 & -\infty < x \leq 1 \\ 2e^{1-x} & 1 < x < \infty \end{cases}$$

Is $f'(x)$ continuous at $x = 1$? Support your answer.

13. Sketch a graph with the following properties

(a)
- $f' < 0$ on $(-\infty, -1)$ and $(2, \infty)$
- $f'(-1)$ is undefined and $f'(2) = 0$
- $f(1) = 0$
- $\lim_{x \to \infty} f'(x) = 0$

(b)
- $f(x) = 0$ at $x = -3$, $x = 2$, and $x = 7$
- $\lim_{x \to -\infty} = \infty$
- $\lim_{x \to \infty} = -5$
- $f'(x) = 0$ at $x = -1$, $x = 4$, and $x = 9$
- $f(10) = -4$