Assignment 7

To be done on a **separate** piece of paper! Please **do not** cram all of your answers onto this worksheet!
Due **Friday 10/31/14** in class.

1. Find and classify the critical points of the following functions.

   (a) $y = x^3 - \frac{9}{2}x^2 - 12x + 1$

   (b) $g(t) = \frac{t}{8} + \frac{1}{2t^2}$

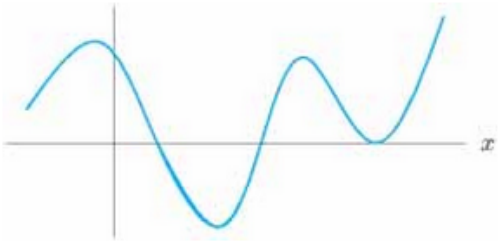
   (c) $v = k\sqrt{\frac{\lambda}{c} + \frac{c}{\lambda}}$, where c and k are positive constants.

2. (a) Find all critical points of the function $f(x) = x^4 - 2ax^2 + b$, where a and b are positive constants.

   (b) Find the values of the parameters a and b if f has a critical point at (2,5).

   (c) If there is a critical point at (2,5), where are the inflection points?

3. Indicate on the graph below approximately where the inflection points of $f(x)$ are if the graph shows (a) the function $f(x)$; (b) the derivative $f'(x)$; (c) the second derivative $f''(x)$. Redraw the graph each time.



4. Find the global maximum and global minimum of the following functions on the given interval. If a global minimum or maximum does not exist, find the best possible upper and lower bound.

   (a) $y = xe^{-x/3}$ on the interval [0,10].

   (b) $y = xe^{-x/3}$ on the interval (0,10).

   (c) $f(x) = x^2 - 2|x|$ on the interval $-3 \leq x \leq 4$.

   (d) $g(x) = x - \ln(x)$ on the interval $(0, \infty)$.

5. The potential energy U of a particle moving along the x-axis is given by

$$U = b\left(\frac{a^2}{x^2} - \frac{a}{x}\right),$$

   where a and b are positive constants and $x > 0$. What value of x minimizes the potential energy?

6. When an electric current passes through two resistors with resistance r_1 and r_2, connected in parallel, the combined resistance R is given by

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2},$$

   where r_1 and r_2 are positive. Assume that r_2 is constant.

   (a) Show that R is an increasing function of r_1.

   (b) Where on the interval $a \leq r_1 \leq b$ does R take its maximum value?