Assignment 5

To be done on a **separate** piece of paper! Please **do not** cram all of your answers onto this worksheet!
Due **Thursday 10/16/14** in class.

You **must show your work** to receive credit. You should *check* your answers online.

1. Find the derivative of the following reciprocal functions.

   (a) $y = \sec(\theta)$
   (b) $y = \csc(\theta)$
   (c) $y = \cot(\theta)$
   (d) $y = \operatorname{sech}(x)$
   (e) $y = \operatorname{csch}(x)$
   (f) $y = \coth(x)$

2. Find the derivative of the following functions. Treat a and b as constants. Simplify.

   (a) $f(\theta) = \cos^2\big(\sin(\theta)\big) + \sin^2\big(\sin(\theta)\big)$
   (b) $f(t) = \sinh^2(8t^3) - \cosh^2(8t^3)$
   (c) $r(y) = \sin\big(a\sin(x) + \cos(bx)\big)$
   (d) $G(\omega) = 2\sin(\omega)\cos(\omega)$
   (e) $y = \cosh(at)\sinh(bt)$
   (f) $U(r) = \operatorname{sech}^2(\sqrt{a}r)$

3. Find and simplify the derivative of $f(x) = \sqrt{\dfrac{1-\sin(x)}{1-\cos(x)}}$.

4. Simplify and find the derivative of $g(t) = \cosh\big(\ln(t)\big)$.

5. (a) Calculate $\dfrac{d^{97}}{d\theta^{97}}\cos(\theta)$.
   (b) Calculate $\dfrac{d^{49}}{dx^{49}}\sinh(2x)$.
   (c) Find an expression for $\dfrac{d^n}{dx^n}\sin(ax)$. *Hint:* write your answer as a piecewise expression, depending on whether n is even or odd. You may also need a factor of $(-1)^{(n-1)/2}$ or $(-1)^{n/2}$.

6. A block of mass m is hanging at rest from a spring attached to the ceiling. If you pull the block down a distance y centimeters from its resting position, the spring will exert a force equal to $-ky$ to bring the block back to its resting position, where k is a constant. When you release the block, it will bounce up and down in simple harmonic motion. Let $y(t)$ be the distance of the block from its resting position in centimeters at time t seconds.

   (a) Write an expression, in terms of $y(t)$, for the velocity and acceleration of the block at time t.

   (b) Newton's second law of motion tells us that the acceleration of the block is directly proportional to the force from the spring, with the constant of proportionality equal to $1/m$, i.e. $a = F/m$. Write Newton's second law as an equation for y, using your expression from (a) for the acceleration.

   (c) Show that the function $y(t) = A\sin\left(\sqrt{\frac{k}{m}}t\right) + B\cos\left(\sqrt{\frac{k}{m}}t\right)$ satisfies your equation in part (b) for any constants A and B. This is the height of the block t seconds after it is released.

   (d) With $y(t)$ from part (c), find the block's initial position and initial velocity, in terms of A, B, m, and k.

   (e) What is the period T of the oscillation?

   (f) Calculate $\dfrac{dT}{dm}$. Interpret the sign ($\pm$) of your result.

7. The cable between two towers of a power line hangs in the shape of the curve

$$y = \frac{T}{w}\cosh\left(\frac{wx}{T}\right),$$

   where T is the tension in the cable at its lowest point and w is the weight of the cable per unit length. This curve is called a *catenary*.

   (a) Suppose the cable stretches between the points $x = -\dfrac{T}{w}$ and $x = \dfrac{T}{w}$. Find an expression for the sag in the cable (i.e. find the difference between the height of the cable at its highest and lowest points).

   (b) Show that the cable satisfies the equation (also derived from Newton's second law!)

$$\frac{d^2y}{dx^2} = \frac{w}{T}\sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$