Assignment 4

To be done on a **separate** piece of paper! Please **do not** cram all of your answers onto this worksheet!
Due **Tuesday 10/14/14** in class.

1. Use the product rule and quotient rule to calculate each derivative. Simplify as much as possible.

   (a) $\dfrac{d}{dt}\big(e^{-2t}(t^2+\sqrt{t^3})\big)$

   (b) $\dfrac{d}{dy}\big(ay^2e^{-by}\big)$

   (c) $\dfrac{d}{d\theta}\left(\dfrac{\theta^2+\sqrt{\theta}}{\theta^2+2\theta-1}\right)$

   (d) $\dfrac{d}{du}\left(\dfrac{e^u-e^{-u}}{e^u+e^{-u}}\right)$

   (e) $\dfrac{d}{dx}\big(f(x)g(x)h(x)\big)$

   (f) $\big(f(x)^2\big)'$

2. Use the chain rule to calculate each derivative. Simplify as much as possible.

   (a) $\dfrac{d}{du}\big(e^{-u^2}\big)$

   (b) $\dfrac{d}{dx}\big(\sqrt{r^2-x^2}\big)$

   (c) $\dfrac{d}{dt}\big((e^{2t}+1)^m\big)$

   (d) $\dfrac{d}{dx}\left[f\Big(g\big(h(x)\big)\Big)\right]$

3. Find the derivative of $f(x)=\left(\dfrac{e^{-x}(1+x^2)}{(1-x^2)^2}\right)^3$. Simplify as much as possible.

4. Use the power rule, product rule, and chain rule to evaluate $\dfrac{d}{dx}\Big(f(x)\big(g(x)\big)^{-1}\Big)$. Write your result as a single, simplified fraction with no negative exponents.

5. (a) For what values of a is the graph of $y = axe^{-ax}$ increasing at $x = 4$?

   (b) Find the value(s) of a so that $f(x) = e^{-ax^2}$ has an inflection point at $x = 2$.

6. It is highly recommended that you use Leibniz notation for this problem.

   (a) i. Expand $(x + y)^2$

   ii. Calculate $\dfrac{d^2}{dx^2}\big(u(x)v(x)\big)$

   (b) i. Expand $(x + y)^3$

   ii. Calculate $\dfrac{d^3}{dx^3}\big(u(x)v(x)\big)$

   (c) i. Expand $(x + y)^4$

   ii. Calculate $\dfrac{d^4}{dx^4}\big(u(x)v(x)\big)$

   (d) Hopefully you've noticed a pattern. The Binomial Theorem tells us how to expand $(x + y)^n$ for any integer n:

$$(x + y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i},$$

   where $\dbinom{n}{i} = \dfrac{n!}{i!(n-i)!}$ are the binomial coefficients. Write an expression for $\dfrac{d^n}{dx^n}\big(u(x)v(x)\big)$. *Hint:* this is an exercise in notation.