Assignment 3

To be done on a **separate** piece of paper! Please **do not** cram all of your answers onto this worksheet!
Due **Wednesday 10/1/14** in class.

1. For (a)-(d), sketch a graph that meets the criteria.

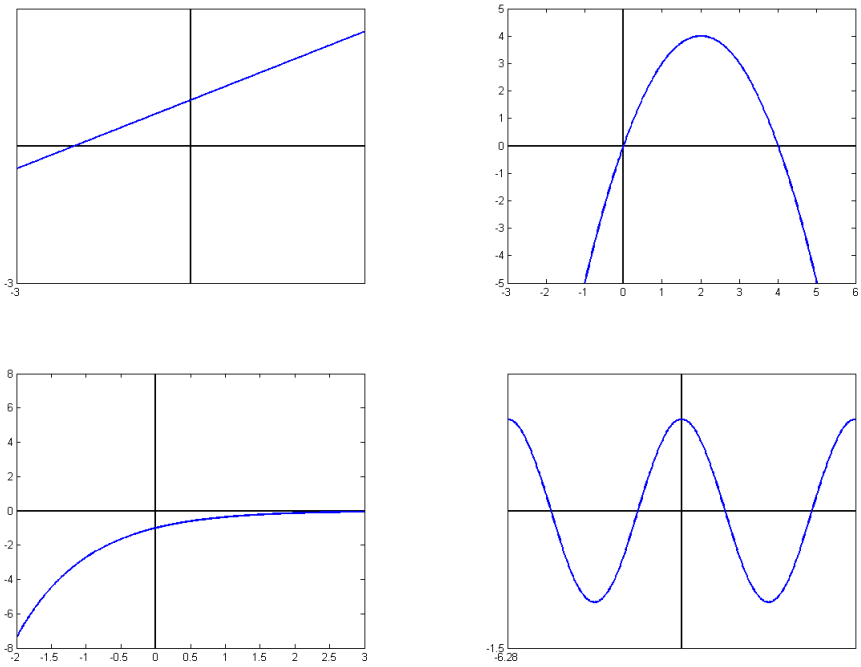
   (a) First and second derivatives are everywhere positive.

   (b) First derivative is positive everywhere, second derivative is negative everywhere.

   (c) First derivative is negative everywhere, second derivative is positive everywhere.

   (d) First derivative is negative everywhere, second derivative is positive on the interval $(-\infty, 3)$ and negative on the interval $(3, \infty)$.

2. Sketch the second derivative of the following functions. *Hint:* start by sketching the first derivative



3. Use the table to answer the following questions.

t	$f(t)$	$g(t)$	$h(t)$	$i(t)$	$j(t)$
5	50	19	14	30	49
10	43	25	21	33	45
15	38	29	28	39	39
20	35	31	35	50	30

(a) Which function has concavity zero?

(b) Which function is decreasing and concave up?

(c) Which function is increasing and concave down?

(d) Use the data for function $j(t)$ to estimate $j''(10)$

4. A continuous function f defined for all x has the following properties:

- f is increasing
- f is concave down
- $f(5) = 2$
- $f'(5) = 0.5$

(a) Sketch a possible graph of $f(x)$

(b) How many zeros does $f(x)$ have?

(c) What can you say about the location of these zeros?

(d) It it possible that $\lim_{x \to -\infty} f(x) = -5$? Explain.

(e) Is it possible that $f'(1) = 1$? Explain.

(f) Is it possible that $f'(1) = 0.25$? Explain.

5. Determine if the following statements are true of false. If it is false, give a counterexample.

(a) If a function is differentiable, then it is continuous.

(b) If a function is continuous, then it is differentiable.

(c) If a function is not continuous, then it is not differentiable.

(d) There exists a function that is continuous on [1,5] but not differentiable at $x = 3$.

(e) *Bonus:* There does not exist a function that is discontinuous at *every* point in its domain.