

Studying the Geomagnetic Polarity Time Scale and Superchrons by Scalar Models and Feature Based Data Assimilation

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Summary

Stochastic models for producing time-series of Earth's magnetic field use paleomagnetic data [1, 6] from the past 2 Myr, and can match mean dipole reversal rates on this time scale. The models are characterized by two components: a deterministic drift coefficient modeling "resolved" dynamics, and a stochastic diffusion term modeling the effects of turbulent fluid motion at Earth's core. The models are suspect over longer time periods, due to significant variation in core dynamics. We propose a means of extending the models' applicability to time periods on the order of 150 Myr by adjusting the diffusion coefficient based on the geomagnetic polarity time scale [2]. This allows the models to produce superchrons: periods of more than 10 Myr with no reversals.

Data

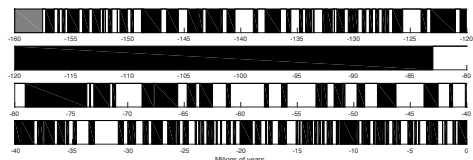


Figure 1: Geomagnetic field polarity over the past 157 Myr. Black denotes the current orientation, white is the opposite [2, 3]. The Cretaceous Normal Superchron occurred 120 Myr ago, and lasted nearly 40 Myr.

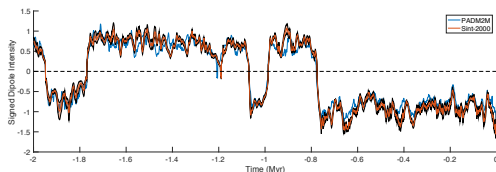


Figure 2: Signed dipole intensity over the past 2 Myr as documented by Sint-2000 and PADM2M data sets [6, 7].

Models

Stochastic differential equation (SDE) models of the dipole are of the form

$$dX = f(X) dt + g(X) dW$$

where $f(X)$ and $g(X)$ are scalar functions, and represent the resolved dynamics of the system, and the effects of turbulence and flow in the core, respectively. W is Brownian motion, with $W(t) - W(s) \sim N(0, t-s)$, and $W(0) = 0$.

B13 Model: The state X represents the state of the dipole, and the system is modeled as a particle in a double-well potential [1]. The coefficients $f(X)$ and $g(X)$ are estimated from Sint-2000 and PADM2M data, which informs the time scale for t .

P09 Model: The state X represents the angle of the dipole, with

$$f(X) = \alpha_0 + \alpha_1 \sin(2X), \quad g(X) = 2\sqrt{|\alpha_1|}$$

and the dipole is calculated as $D = R \cos(X + X_0)$. The same parameters as in [5] are used below. R is chosen to scale the dipole amplitude with the Sint-2000 data. Figure 3 shows typical simulation results from these models.

Motivation

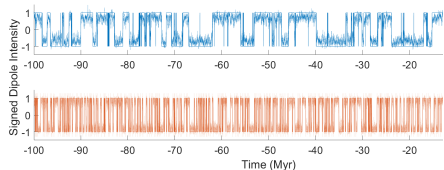


Figure 3: Both the B13 and P09 models fail to capture superchrons. This implies a misrepresentation of the core dynamics in the model. Our proposed solution is to introduce a multiplicative scaling function for the noise,

$$dX = f(X) dt + \alpha(t)g(X) dW$$

and to estimate the scaling from the geomagnetic polarity time scale.

Methods

In typical data assimilation, parameters are estimated to fit numerical models to measured data. This approach is not feasible for this problem because the data are the sign of the state. We instead preprocess the data to produce a feature: the mean chron duration (MCD) estimated over time. We base our data assimilation on this feature, approximating the distribution of reversals, as opposed to the exact reversal times. We first measure the relationship between the noise scale α and the MCD. We then run the models with constant values for α , and calculate the resulting MCD.

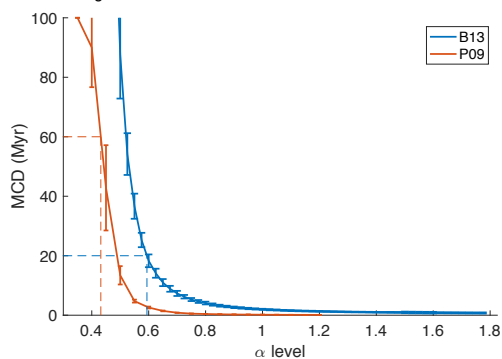


Figure 4: MCD as a function of the noise scale.

Results

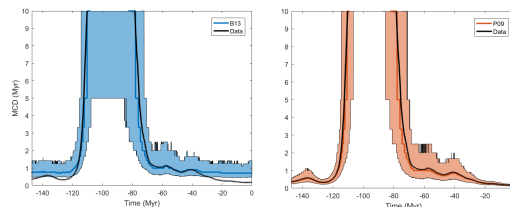


Figure 5: 500 independent simulations of the B13 and P09 models using the estimated noise scaling. Both the B13 (left) and P09 (right) models approximate the MCDs observed in the data from figure 1. The error bars consist of the 2.5% and 97.5% quantiles

Results (cont.)

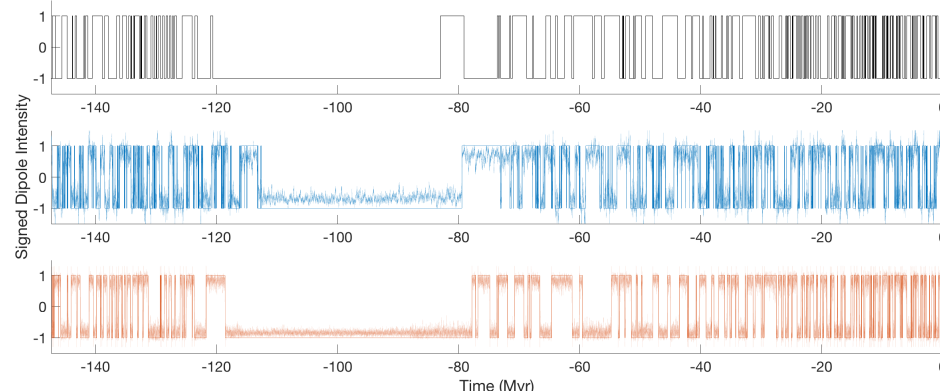


Figure 6: Simulations of B13 (middle) and P09 (bottom) with the noise scaling as estimated from the geomagnetic polarity time scale. Note that both models exhibit superchrons and that the superchrons occur (roughly) at the right time when compared with the data (top, and figure 1). This is in stark contrast to simulations with constant noise scale when no superchrons occur (see figure 3).

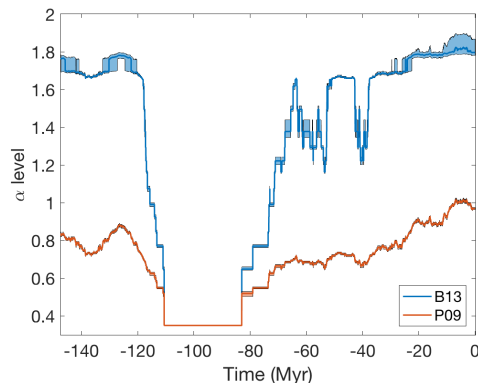


Figure 7: Noise scaling estimated from the geomagnetic polarity time scale for the B13 and P09 models. The estimation was performed by inverting the geomagnetic polarity time scale using the alpha-MCD dependency computed from the models (see figure 4).

References

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Discussion

The simplified B13 and P09 models are derived by drastically simplifying the magnetohydrodynamics (MHD) equations that govern the magnetic field. The noise terms and their scaling thus have immediate physical meaning and in particular summarize the effects of turbulent fluctuations in the core flow. By studying the temporal variation of the noise term one can thus derive statements about dynamics at the core and what mechanisms lead to superchrons. A detailed study of the physical interpretation of our results is currently in the making. For a related study using the B13 and P09 model, see [4].

Acknowledgements

This work was supported by National Security Technologies, LLC, under Contract No. DE-AC52-06NA25946 with the U.S. Department of Energy and supported by the Site-Directed Research and Development program. DOE/NV/25946--3085. JA and MM were supported in part by the National Science Foundation under grant DMS-1619630. JA was supported by the Department of Mathematics at the University of Arizona. MM acknowledges the support of an Alfred P. Sloan Research Fellowship.