

Introduction to Quadratic Functions

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October 19, 2009

Motivation

Why does one go into business?

What is the goal of a person running a business?

On Wednesday, when we conclude this section, we will see how to accomplish this goal under certain assumptions.

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Definitions

A **quadratic function** is a function of the form

$$f(x) = ax^2 + bx + c$$

for some fixed numbers a, b, c with $a \neq 0$.

For example: $f(x) = 2x^2 - 12x + 10$ is a quadratic function.

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Standard Form

Every quadratic function can be written in **standard form**:

$$f(x) = a(x - h)^2 + k.$$

The goals today are:

- 1 Figure out how to graph a quadratic function written in standard form by hand, and
- 2 figure out how to write a given quadratic function written as $f(x) = ax^2 + bx + c$ in standard form.

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Graphing Quadratic Functions

A quadratic function in standard form $y = a(x - h)^2 + k$ can be realized as a sequence of transformations of the base graph $y = x^2$. Use transformations of the graph $y = x^2$ to graph the following quadratic functions:

- 1 $f(x) = x^2 - 3$
- 2 $f(x) = -(x - 2)^2$
- 3 $f(x) = 3(x + 1)^2 - 2$
- 4 $f(x) = -2(x - 1)^2 + 1.$

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Discussion: Features of Quadratic Functions

For the functions that you graphed, determine:

- 1 Domain
- 2 Range
- 3 Intercepts
- 4 Max/Min?
- 5 Symmetry?

Finding Standard Form

Given some quadratic function $f(x) = ax^2 + bx + c$, how can we rewrite the function in standard form?

Typically one would complete the square.

Another way to do this uses the axis of symmetry. First we will do this for the function

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How to Find Standard Form

Bottom line, using the axis of symmetry, the quadratic function $f(x) = ax^2 + bx + c$ is the same as

$$y = a \left(x + \frac{b}{2a} \right)^2 + f \left(\frac{b}{2a} \right).$$