

Population Dynamics

Algebra 5/Trig

Spring 2010

Instructions: There are none! This contains background information and some suggestions for your project. Feel free to make changes and ask different questions if you want, subject to your teacher's approval.

1 Background

Using calculus and differential equations, population growth can be modeled by a function of the form

$$P(t) = \frac{KP_0e^{rt}}{K + P_0(e^{rt} - 1)}$$

where t is time, $P(t)$ the population at time t , P_0 the population at time $t = 0$, r the growth rate, and K is something called the “carrying capacity.” Before proceeding, you may wish to review properties of exponential functions and graphs in section 5.1 of your textbook.

2 Some Questions

1. According to problem 60 in section 5.1 of your textbook, the population of a town grows according to the model

$$P(t) = 2500e^{.0293t}.$$

Graph this function and explain why it is an unrealistic model.

2. In the model

$$P(t) = \frac{KP_0e^{rt}}{K + P_0(e^{rt} - 1)}$$

, take $r = 1$, $K = 2$ and $P_0 = 2$. Graph this function. Explain why this graph looks appropriate for population growth.

3. This function can be rewritten as:

$$P(t) = \frac{KP_0}{(K - P_0)e^{-rt} + P_0}.$$

If you are feeling ambitious, try to justify this. Otherwise, taking this as given, keeping in mind that $r > 0$, what happens as $t \rightarrow \infty$. Use this to interpret the meaning of the constant K and why it is called the “carrying capacity.”

4. Look up the current growth rate of the world population. Find out how much time it took for the population to grow from 5 billion to 6 billion, and use this to make an estimate for the constant K . Find today's world population for P_0 and graph the resulting function. Estimate when the world population will be within one-million of the carrying capacity.