

Pascal's Pyramid

Algebra 5/Trig

Spring 2010

Instructions: There are none! This contains background information and some suggestions for your project. Feel free to make changes and ask different questions if you want, subject to your teacher's approval.

1 Background

In Chapter 11, you studied the binomial theorem. This theorem tells you how to expand $(x + y)^n$ using binomial coefficients:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}.$$

The binomial coefficients can be computed using Pascal's triangle.

Now we would like to expand $(x + y + z)^n$. The way to do this is to use trinomial coefficients, which can be generalized to multinomial coefficients. Note that we can change the notation for the binomial coefficient as

$$\binom{n}{k_1, k_2} = \frac{n!}{k_1!k_2!}$$

where we require $k_1 + k_2 = n$. A trinomial coefficient would like like

$$\binom{n}{k_1, k_2, k_3} = \frac{n!}{k_1!k_2!k_3!}$$

where we require $k_1 + k_2 + k_3 = n$. More generally, a multinomial coefficient is defined to be

$$\frac{n!}{k_1!k_2!\cdots k_m!} = \frac{n!}{k_1!k_2!\cdots k_m!}$$

where we require $k_1 + k_2 + \cdots + k_m = n$. These multinomial coefficients can be used to expand powers of sums using the multinomial theorem and they can also be used to solve counting problems. Trinomial coefficients can be computed using Pascal's pyramid, which is a generalization of Pascal's triangle.

2 Some Questions

1. Compute the following multinomial coefficients: $\binom{5}{1,2,2}$, $\binom{7}{2,3,2}$, $\binom{10}{2,2,3,3}$, $\binom{12}{2,3,3,4}$.

2. Read the copied text on multinomial coefficients. Pay attention to:

- (a) The formula for the multinomial coefficients,
- (b) Pascal's pyramid,
- (c) The multinomial theorem, and
- (d) Applications to counting (at the end).

Try to understand how to construct Pascal's pyramid and the statement of the multinomial theorem. Ignore the formula labeled "symmetry" and the one labeled "addition." Also, ignore the proof of the multinomial theorem.

3. Construct a model of Pascal's pyramid.

4. Use the multinomial theorem to expand $(x + y + z)^2$ and $(x + y + z)^3$.

5. Do problems 9, 10 and 11 in the exercises in the copied text. Ignore the other exercises, they are more theoretical.

Note that the copied text comes from *Combinatorics and Graph Theory*, second edition, by John M. Harris, Jeffrey L. Hirst, and Michael J. Mossinghoff; published by Springer in their Undergraduate Texts in Mathematics Series in 2008.