

Old Colony Trust Co. v. Commissioner

Algebra 5/Trigonometry

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In the case Old Colony Trust Co. v. Commissioner, 279 U.S. 716 (1929), the Supreme Court was asked to determine whether the payment by an employer of an employee's income is itself taxable income. An interesting historical footnote, the opinion in this case was written by Chief Justice William Howard Taft, who so far is the only person in U.S. history to serve as U.S. President **and** on the Supreme Court.

The court decided that the answer to the question is yes, if an employer pays your income taxes for you then the payment constitutes taxable income. The attorneys for the Old Colony Trust Co. tried to argue that the payment of the additional tax will constitute further income which will then have to be taxed, and so on resulting in an absurdity. The court brushed aside this argument without much analysis. In this worksheet, we will analyze this argument mathematically.

To make things simple, suppose the income paid to the employee is \$100,000.00 and that there is a flat tax rate of 25 %, which we will write as $1/4$.

1. In the first instance, we have the amount of taxes owed on the \$100,000 of income. Write this amount.
2. Next we have the amount of taxes owed on income taxes you computed above. Write this amount.
3. Next we have the amount of taxes on the taxes on the original taxes paid. These are the taxes owed on the taxes you just computed. Write this amount.
4. Keep going and look for a pattern in the sequence. What type of sequence is it? Write a formula for the n^{th} term. Our goal is to compute the sum of the terms of this (infinite) sequence.

5. The sum of the first n terms of a geometric sequence is

$$\sum_{k=1}^n a_1 r^{k-1} = a_1 \left(\frac{1 - r^n}{1 - r} \right).$$

We will try to figure out what happens when $n \rightarrow \infty$.

- (a) Suppose $|r| < 1$. What happens to r^n as $n \rightarrow \infty$? You may want to compute some powers of some examples such as $1/2$ or $-1/3$ in order to help you answer this question.
- (b) Suppose $|r| > 1$. What happens to r^n as $n \rightarrow \infty$? You may want to compute some powers of some examples such as 2 or -3 in order to help you answer this question.
- (c) What happens to r^n as $n \rightarrow \infty$ if $r = 1$ or $r = -1$?
- (d) Use your responses to the three questions above to find a formula for the **infinite** sum

$$\sum_{k=1}^{\infty} a_1 r^{k-1}$$

and state the r 's for which the formula is valid.

6. Use the formula you just found to compute the total amount of taxes an employer would have to pay in order to pay an employee \$ 100,000.00 net of taxes. What do you think of Old Colony's argument that this result is absurd?