

Music and Trig: Why Does Autotune Sound So Funny?

Algebra 5/Trig

Spring 2010

Instructions: There are none! This contains background information and some suggestions for your project. Feel free to make changes and ask different questions if you want, subject to your teacher's approval.

1 Background

A sound wave can be modeled with a sine function of the form:

$$y = a \sin bt$$

where t is time. The coefficient a is the amplitude, which can be thought of like volume. The number b changes the period and frequency, which in turn adjusts the pitch. The period is $2\pi/b$ and the frequency is the reciprocal, $b/2\pi$. Frequency is measured in cycles per second, also called hertz. Every pitch is represented by a unique frequency. For example, the note middle A, the first A above middle C corresponds, to a frequency of 440 hertz. This is often called "A440" and is the note to which pianos and orchestras are frequently tuned.

You may wish to review section 6.4 in the textbook in order to see what the graph of a sine function looks like and how the parameters a and b affect the graph.

When one sings, we can model the vibrations with a fancy "partial differential equation" called the wave equation. The solutions to the wave equations are **sums** of sine functions, not just a single sine function. These sums are typically infinite, and called **Fourier series**. We will only consider finite approximations. The term with the largest amplitude corresponds to the main pitch one hears, and all of the other terms have pitches that correspond to multiples of this fundamental frequency and are called overtones.

When a singer uses the program Autotune, the sound produced only involves the term with the largest amplitude. Since there are no overtones, the resulting sound has a synthetic quality.

2 Some Questions

One way of presenting your project would be to find a way of plugging a microphone or a guitar into a computer and have the computer display graphs of sound waves.

1. Assume the amplitude is 1 and write down the sine function for middle A. To do this, you need to find the parameter b using the fact that the frequency is 440 Hz.
2. An **octave** above a note is defined to be the pitch whose frequency is twice the original frequency. How would you define an octave below a note? How about n octaves above a note? Can you write down a sequence for all notes labeled A?
3. Jim Gillette, from the (awful) hair metal band Nitro, claimed in the late 1980's that he had a vocal range of 7 octaves. Suppose that this is true, and suppose that middle A is in the middle of his range. Then he can hit an A three octaves below and three octaves above middle A. Write down the sequence of frequencies of the A notes that Jim Gillette can hit, graph the sequence, and decide if you think this looks realistic. To hear Jim display his range, you can search "Jim Gillette hits an insane scream" on youtube, and if you want to see how ridiculous his band Nitro was, search "Nitro freight train" on youtube.
4. Look up "homemade oscilloscope bohemian rhapsody" on youtube to see a graph of sound waves produced by the classic Queen song. For some reason, this graph is vertical. Try to observe the amplitude and frequency changes in the graph.
5. Look up "sawtooth function" on wikipedia and try to understand the Fourier series that approximates this function, the animation showing how the sum converges to the function and listen to the audio sample on the page. Note that the term "harmonics" refers to the terms in the Fourier series.
6. Graph the following functions and compare: the first is the graph of a singer singing A440 through Autotune, and the second is a model of a possible approximation of the actual sound when a singer sings A440. We set the amplitude to one, to keep things simple. Compare the graphs.

(a) $y = \sin(880\pi x)$

(b) $y = \sin(880\pi x) + \frac{\sin(1760\pi x)}{2} + \frac{\sin(2640\pi x)}{3} + \frac{\sin(3520\pi x)}{4}$