The Mean, Variance and Standard Deviation of a Discrete Random Variable

Victor I. Piercey

November 30, 2009

The Mean of a Discrete Random Variable

If you have a discrete random variable X with the following probability distribution:

Value of X	x_1	x_2	$\cdots$	x_n
Probability	p_1	p_2	$\cdots$	p_n

the **mean** of X is the sum of the products $x_i p_i$:

$$\mu_X = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n = \sum x_i p_i.$$

This is sometimes called the **expected value**.

Example

The distribution for X where X is the number of rooms in an owned house in San Jose, CA is as follows:

Rooms	1	2	3	4	5
Probability	0.003	0.002	0.023	0.104	0.210
Rooms	6	7	8	9	10
Probability	0.224	0.197	0.149	0.053	0.035

Find the mean μ_X.

Centers and Spreads

The mean measures where the distribution is centered.

The variance and standard distribution, which we will compute next, measures how the distribution is spread out.

The Variance and Standard Deviation of a Discrete Random Variable

If you have a discrete random variable X with the following probability distribution:

Value of X	x_1	x_2	$\cdots$	x_n
Probability	p_1	p_2	$\cdots$	p_n

with mean μ_X, then the **variance** of X is

$$\sigma_X^2 = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + \cdots + (x_n - \mu_X)^2 p_n = \sum (x_i - \mu_X)^2 p_i.$$

The **standard deviation** σ_X is the square root of the variance.

Example

The distribution for X where X is the number of rooms in an owned house in San Jose, CA has mean $\mu_X = 6.284$ and is as follows:

Rooms	1	2	3	4	5
Probability	0.003	0.002	0.023	0.104	0.210
Rooms	6	7	8	9	10
Probability	0.224	0.197	0.149	0.053	0.035

Find the variance σ_X^2 and the standard deviation σ_X.

Assignment

Page 486: 7.23,7.25 and 7.30

Page 491: 7.32, 7.33

Due Wednesday