Matrices and Computer Graphics

Algebra 5/Trig

Spring 2010

Instructions: There are none! This contains background information and some suggestions for your project. Feel free to make changes and ask different questions if you want, subject to your teacher's approval.

1 Background

In this project, we will simulate the rotation of a pyramid in computer graphics by rotating its vertices.

Suppose we can have a computer graphics program sketch a pyramid by specifying coordinates in three-space for the vertices. The coordinates are x, y and z. We can form a column matrix (also called a vector) with the coordinates for each vertex:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

To rotate the vertex about the z-axis by an angle θ we multiply this vector by the matrix:

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

To do the same about the x-axis we use the matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}.$$

To do this about the y-axis, use:

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}.$$

To simulate a camera panning around a pyramid (where the z-axis points up), one can do a successive but finite number of rotations about the z-axis by very small angles.

You may want to review matrix multiplication in section 10.2 of your textbook.

2 Some Questions

1. Write down the matrix for a rotation about each of the three axes by 45 degrees.

2. Sketch a pyramid in 3-space by setting the vertices at the coordinates: $(1, 1, 0), (-1, 1, 0)$, $(1, -1, 0)$, $(-1, -1, 0)$ and $(0, 0, 1)$.

3. Use the matrix for rotating the pyramid about the z-axis by 45 degrees to find the coordinates of the vertices of the rotating pyramid. Sketch this pyramid.

4. Now take the transformed coordinates and multiply the vectors by the matrix for a 45-degree about the x-axis. Sketch the result. Effectively, you have transformed the original vectors by the product of the two matrices. Interpret the product of the matrices geometrically, based on this example.

5. What if instead of having the second rotation about the x-axis, we did another rotation about the z-axis by 30 degrees. Interpret the product of these matrices.

6. Multiply the matrix for rotation about the z-axis by an angle θ by the matrix for rotation about the z-axis by another angle φ. This is the matrix for rotation about the z-axis by an angle $\theta + \varphi$. Use the addition of angles formulae for sine and cosine to justify this interpretation.

7. If you are familiar with computer graphics software, try to use the math above to simulate a pan of a camera about a pyramid.

8. Compute the determinant of the three rotation matrices given above. What does this determinant tell you? You may want to review determinants in section 10.4 of your textbook.