Introduction to Logarithms

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Motivation: My Senior Thesis

In my senior thesis, I wanted to estimate productivity in the Russian defense sector in the mid-1990's.

In particular I wanted to test for "Cobb-Douglas" production technology:

$$Q = AL^{\beta_1} K^{\beta_2}$$

where Q is output, L is the amount of labor input, K is the amount of capital input, and A, β_1, β_2 are productivity parameters.

I had data for Q, L and K and wanted to run regression.

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Finding Inverse Functions

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Exponential Functions

The equation $y = 2^x$ defines a function. We can plug in values and graph this function if we like.

More generally, if $0 < b < 1$ or $b > 1$, then the equation $y = b^x$ defines a function, called the **exponential function** of base b.

The most common bases are $b = \mathrm{e}$ and $b = 10$.

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Logarithms

There **is** an inverse function, but ordinary algebra will not help find it.

Instead, we invent notation to define the inverse function:

$$y = \log_2(x).$$

By definition, $\log_b(x) = y$ means $b^y = x$.

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Practice Computing Some Logarithms

For practice, let's compute the following:

1. $\log_2(4)$
2. $\log_3(27)$
3. $\log_7\left(\frac{1}{7}\right)$
4. $\log_b(1)$ for any base b

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Special Logs

The base $b = \mathrm{e}$ occurs frequently in nature, so the logarithm with base e is called the natural log and it is denoted $\ln(x)$.

The base $b = 10$ is very common, so it is called the common log and is denoted $\log(x)$, with the base suppressed.

These are the only logarithms that can be computed on your calculator (without using the change of base formula).

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Properties of Exponents

The reason logarithms are so important is because of their properties, which come from properties of exponents.

The following are the properties of exponents:

1. $b^{n+m} = b^n b^m$
2. $b^{n-m} = \dfrac{b^n}{b^m}$
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Properties of Logarithms

Since logarithms are inverses of exponential functions, one can invert the properties of exponents to obtain the following properties:

1. $\log_b(MN) = \log_b(M) + \log_b(N)$
2. $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$
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Practice

Use the properties of logarithms to expand the following:

1. $\log_2(xy^2)$
2. $\ln\left(\frac{x^2y^3}{\sqrt{z}}\right)$
3. $\log\left(\frac{1}{x^6}\right)$.

Use the properties of logarithms to write the following as a single logarithm with no coefficient:

1. $\log_3(x) - 2\log_3(y)$
2. $4\ln(x) + 2\ln(y)$
3. $\frac{1}{2}\log(x) - \frac{1}{2}\log(y)$.

Cobb-Douglas Production Estimation

Recall that I wanted to run a regression with the non-linear model:

$$Q = AL^{\beta_1} K^{\beta_2}.$$

To make this linear, I hit both sides with a logarithm and use the properties to obtain:

$$\ln(Q) = \ln(A) + \beta_1 \ln(K) + \beta_2 \ln(L).$$

This is now a linear model (in two input variables) with intercept $\ln(A)$ and slopes β_1 and β_2. I can now run regression.

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