Testing Kepler's Third Law

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In this activity, we will use linear regression on our calculators to test Kepler's Third Law of Planetary Motion.

Kepler's Laws of Planetary Motion state the following:

1. The orbit of every planet is an ellipse with the sun as a focus.
2. A line joining the planet and the sun sweeps out equal areas in equal time.
3. The square of the period of the orbit is equal to the cube of the semi-major axis of its orbit.

If P is the period and x is the semi-major axis, the third law states that

$$P^2 = x^3.$$

We will use data to check for evidence that this is true. The semi-major axis is measured in Astronomical Units (AU) (the semi-major axis of Earth's orbit) and the period is measured in Earth-years.

Before we begin, you should clear your lists and clear Y_1.

1. Insert the following data into two lists as L_1 and L_2 (respectively) in your calculator:

Planet	**Semi-Major Axis** x	**Period** P
Mercury	0.39	0.24
Venus	0.72	0.61
Earth	1.00	1.00
Mars	1.52	1.88
Jupiter	5.20	11.86
Saturn	9.54	29.46
Uranus	19.19	84.01
Neptune	30.06	164.79
Pluto	39.53	248.54

2. Use the StatPlot menu to set your calculator to graph a scatterplot. Adjust your window appropriately and graph. Does it appear that there is a correlation between semi-major axis and period? Does it look linear?
3. To make this look linear, make list $L_3 = \log(L_1)$ and $L_4 = \log(L_2)$. To do this, move the cursor to the top of the table and highlight L_3 and type in $\log(L_1)$ and likewise for L_4.

4. Use the StatPlot menu to set your calculator to graph a scatterplot of L_3 and L_4. Adjust your window appropriately and graph. Does the relationship look linear?

5. Now let's perform linear regression. Hit the stat button, move the cursor to the right to highlight the "calc" menu, move the cursor down to "LinReg $(ax + b)$" and hit enter. Using the 2^{nd} key, enter L_3, comma, L_4, comma. Then hit the vars button, move the curser left to select "Y-VARS," select function and hit enter, select Y_1 and hit enter. Hit enter again. What is the correlation coefficient?

6. Press the "graph" button to see the the regression line.

7. Now we will undo the log transformation:

   (a) We have $\log(P) = a\log(x) + b$.

   (b) Use a log property to obtain $\log(P) = \log(f(x)) + b$.

   (c) Raise both sides as exponents of 10. What is $10^{\log(P)}$?

   (d) Use properties of exponents to simplify $10^{(\log(f(x))+b)}$.

8. Does this relationship look like Kepler's third law?

9. Finally, suppose an asteroid is found between Mars and Venus and has an orbit with semi-major axis $x = 2$ AU. Based on our model, predict how long a year on this asteroid lasts.