

Inverse Functions

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1 One-to-One Functions

A one-to-one function is a function that satisfies the INVERSE NO-AMBIGUITY CONDITION: for every output, there is exactly one input. If a function f is one-to-one, then it has an inverse that is denoted by f^{-1} .

The following tables represent functions whose inputs are x and whose outputs are y . For each table, answer the following questions: (i) is f one-to-one, and if not give an example of an output that comes from more than one input, and (ii) does the table determine x **as a function of** y ? If so, that function is the inverse function.

x	-2	3	6	7	143
y	12	23	-19	-69	143

x	-2	3	6	7	143
y	-1	97	0	97	1

2 Inverse Functions

For each of the following equations, solve for x . Does the result define x **as a function of** y ?

1. $y = \sqrt{x}$

2. $y = 3x - 8$

3. $y = 2x^2 - 5$

4. $y = \sqrt[3]{2x - 7}$

5. $y = \frac{1}{x}$

6. $y = |x|$

7. $y = x + 2$

8. $y = 3x$

3 Inverse Function Notation

If f is a one-to-one function, its inverse function is denoted with f^{-1} . This is **NOT** an exponent! It does **NOT** mean $1/f$. It means nothing more and nothing less than the inverse function for f . It is used in the same way f is used.

1. The function $f(x) = 5x - 9$ is one-to-one. What is $f^{-1}(8)$?

2. If the function f is one-to-one, what is $f(f^{-1}(8))$?

3. If f is a one-to-one function and $f(2) = 5$, what is $f^{-1}(5)$?

4. ★ Suppose f is one-to-one and has its domain given by $[1, \infty)$ and its range given by $(-\infty, 5]$. What is the domain and what is the range of f^{-1} ?