Introduction to Exponential Functions

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In an exponential expression a^n, the number a is the **base** and n is the **exponent**

The **exponential function** with **base** a is the function

$$f(x) = a^x.$$

Transformations of a^x are also exponential functions.

The only allowed bases are $0 < a < 1$ and $a > 1$. Why?

What do the following mean?

1. 2^3
2. 2^{-3}
3. $2^{3/2}$
4. 2^{π}

An Observation

Note that $2^{-x} = \left(\frac{1}{2}\right)^x$.

In connection with this observation, note that the graph of $\left(\frac{1}{2}\right)^x$ is the same as the graph of 2^x reflected over the y axis.

In general, because of the properties of exponents, two or more exponential functions that look different may be equal.

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Review: Properties of Exponents 1

Consider $2^2 \cdot 2^3$:

$$2^2 \cdot 2^3 = (2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 2^{2+3}.$$

More generally:

$$a^n a^m = a^{n+m}.$$

Using this law:

$$a^{n-m} = a^n a^{-m} = a^n \left(\frac{1}{a^m}\right) = \frac{a^n}{a^m}.$$

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Example: # 12

Use the properties of exponents to determine which functions (if any) are the same:

- $f(x) = 4^x + 12$
- $g(x) = 2^{2x+6}$
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Assignment

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