Hyperbolic Trigonometry

Algebra 5/Trig

Spring 2010

Instructions: There are none! This contains background information and some suggestions for your project. Feel free to make changes and ask different questions if you want, subject to your teacher's approval.

1 Background

If we set $x = \cos\theta$ and $y = \sin\theta$, then by letting θ vary between 0 and 2π we trace out the unit circle. This is because

$$\cos^2\theta + \sin^2\theta = 1$$

which translates to $x^2 + y^2 = 1$.

We can trace out a hyperbola using **hyperbolic** cosine (written cosh and pronounced like it looks) and **hyperbolic** sine (written sinh and pronounced like "synch"). There are explicit formulas for these functions:

$$\begin{aligned} \cosh t &= \frac{\mathrm{e}^t + \mathrm{e}^{-t}}{2} \\ \sinh t &= \frac{\mathrm{e}^t - \mathrm{e}^{-t}}{2}. \end{aligned}$$

You can then form hyperbolic tangent, secant and cosecent as:

$$\begin{aligned} \tanh t &= \frac{\sinh t}{\cosh t} \\ \mathrm{cosech} t &= \frac{1}{\sinh t} \\ \mathrm{sech} t &= \frac{1}{\cosh t} \\ \mathrm{coth} t &= \frac{1}{\tanh t}. \end{aligned}$$

The definitions for $\cosh t$ and $\sinh t$ are not arbitrary. Given any function $y = f(x)$ (with a domain that is symmetric about the origin), we can form the **even part** $f_{\text{even}}(x)$ and **odd part** $f_{\text{odd}}(x)$ of f by:

$$\begin{aligned} f_{\text{even}}(x) &= \frac{f(x) + f(-x)}{2} \\ f_{\text{odd}}(x) &= \frac{f(x) - f(-x)}{2}. \end{aligned}$$

The even part is an even function and the odd part is an odd function, and the two functions add up to the original function f. You can review even and odd functions in section 2.3 of your textbook.

The hyperbolic trig functions obey identities similar to the trig identities. The Pythagorean identity for hyperbolic trig functions is:

$$\cosh^2 t - \sinh^2 t = 1.$$

Because of this identity, if we set $x = \cosh t$ and $y = \sinh t$ and let t vary over all real numbers, we get one-half of the graph of the equation $x^2 - y^2 = 1$, which is a hyperbola.

2 Some Questions

1. Graph the functions $y = \cosh x$ and $y = \sinh x$. Can you see that the first is an even function and the second is an odd function?

2. Verify that $\mathrm{e}^x = \cosh x + \sinh x$.

3. Find explicit formulae for the other four hyperbolic trig functions.

4. Use the parametric mode of your calculator to graph $x = \cosh t, y = \sinh t$ and see that we get half of a hyperbola. Can you find a way to obtain the other half?

5. Verify the identity
$$\cosh^2 t - \sinh^2 t = 1.$$
To do this, use the explicit formulae along with properties of exponents to compute $\cosh^2 t$ and $\sinh^2 t$, then subtract to obtain 1. You can review properties of exponents in section P.2 of your textbook.

6. Look up other hyperbolic trig identities, such as double angle formulas and angle sum formulas. If you are feeling saucy, try to verify them.