General Probability Rules

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Goals

You already know the **rules of probability**:

1. $0 \leq P(A) \leq 1$ for any event A,
2. $P(S) = 1$,
3. If A and B are disjoint ($A \cap B = \varnothing$):

$$P(A \cup B) = P(A) + P(B),$$

4. $P(A^c) = 1 - P(A)$ for any event A,
5. If A and B are independent events, $P(A \cap B) = P(A)P(B)$.

Our goal today is to generalize the third rule to the cases where (i) we have more than two disjoint events, or (ii) A and B are not disjoint.

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General Addition Rule for Disjoint Events

If A,B and C are disjoint events in the sense that no two have any outcomes in common, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C).$$

This holds for any number of disjoint events.

Proof?

General Addition Rule for Unions of Two Events

What if the events A and B are not disjoint ($A \cap B \neq \varnothing$)? What is $P(A \cup B)$?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Proof?

$\star$ Challenge: Prove that: $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$.

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Example 1

Emily applied to the University of Arizona and Arizona State University. She thinks that the probability of getting into UA is 0.4, the probability of getting into ASU is .5, and the probability of getting into both is .2.

1. Make a Venn diagram marked with the given probabilities.
2. What is the probability that neither university admits Emily?
3. What is the probability of getting into ASU but not UA?

Example 2

Say that a household is “prosperous” if its annual income exceeds \$100,000.

Say that a household is “educated” if the head of household completed college.

Let A be the event that a household is prosperous and B be the event that a household is educated.

Example 2

Suppose $P(A) = 0.138$, $P(B) = 0.261$ and $P(A \cap B) = 0.082$.

Draw a Venn diagram and indicate each of the following events in your diagram along with their respective probabilities. Finally, describe each event in words.

1. A and B.
2. A and B^c.
3. A^c and B.
4. A^c and B^c.

Assignment

Page 441, problems 6.66, 6.69 and 6.70.
Page 446, problems 6.72 and 6.78.