

# Finance

Algebra 5/Trig

Spring 2010

**Instructions:** There are none! This contains background information and some suggestions for your project. Feel free to make changes and ask different questions if you want, subject to your teacher's approval.

## 1 Background

In this project we will use mathematical models to value streams of payments. You should review compound interest in section 5.1 of your textbook and geometric sequences and series in section 11.3 of your textbook.

The basic idea here is the time value of money. Because money can earn interest in an account, a fixed amount of dollars today is worth more money in the future. This is called future value. Similarly, in order to obtain a fixed amount of money at a certain point in the future, some number of dollars must be invested today. This is called present value. Finding the present value of some investment is called “discounting to present value.”

The basic tool is compound interest. We will assume all of our compounding is monthly. We will let  $i$  denote the monthly interest rate. If  $r$  is the annual interest rate,  $i = r/12$  (in fact, the letter  $r$  may be useful in another role). In addition, we will let  $t$  denote time measured in months, not years. This way, the compound interest formula tells us that the **future value** of  $P$  dollars invested at interest rate  $i$  for  $t$  months is

$$A = P(1 + i)^t.$$

Similarly, solving for  $P$ , we see that the **present value** of  $A$  dollars of money  $t$  months in the future under a monthly interest rate  $i$  is

$$P = \frac{A}{(1 + i)^t}.$$

The questions below will guide you in determining how to find the future value and present value of a sequence of payments. If we are receiving the payments, this arrangement is called an **annuity**. We can also suppose we are making the payments. For example, the payments could be the minimum payments on a credit card or payments on a car loan.

## 2 Some Questions

1. Suppose we have a sequence of payments of \$  $P$  for  $n$  months and the monthly interest rate is  $i$ . For each payment, determine the future value and write the results as a sequence. What

type of sequence is this? Find the sum of these  $n$  terms with an appropriate formula. This is the future value of this stream of payments.

2. To find the present value of this stream of payments, we can take the future value from the previous question and consider it as a single future sum of money that we can then discount to present value.
3. Consider a stream of payments of \$ 50 a month for 36 months with a 1.5 % monthly interest rate. Calculate the present and future values.
4. Suppose you owe a balance of \$ 800.00 on a credit card. If your monthly interest rate is 2 % and you make the minimum payment of \$ 10 per month, what is the balance on your credit card after one year? To calculate this, first compute the balance if you made no payments - this is the future value of the \$ 800.00. Next compute the future value of your stream of \$ 10 payments for 12 months. The difference will be the remaining balance on your credit card. What is happening in this scenario?
5. New federal laws passed by the current Congress and signed by President Obama require that credit card companies cannot adjust the minimum payment so that the balance of your credit card account increases if you are not charging more money. Adjust your calculations below to write down an equation whose solution is the smallest possible minimum monthly payment under the new law, and solve this equation.