

The Mean, Variance and Standard Deviation of a Random Variable: Coin Tossings

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1. Suppose you toss a coin 4 times and X is the random variable whose value is the number of heads in an outcome. Recall we have the following probability distribution:

Value of X (number of heads)	0	1	2	3	4
Probability	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$
$x_i p_i$					
$(x_i - \mu_X)^2 p_i$					

- (a) To fill in the third row, multiply each possible value of X by the respective probability.
 - (b) Add the values in the third row. This is the **mean** of the random variable X , written μ_X .
 - (c) To fill in the fourth row, for each possible random variable X , subtract the mean μ_X and square the result. Multiply the value by the respective probability. Enter this result in the fourth row.
 - (d) Add the values in the fourth row. This is the **variance** of the random variable X , written σ_X^2 .
 - (e) As you might expect, the **standard deviation** of the random variable σ_X is the positive square root of the variance.
2. Repeat the same process to find the mean, variance and standard deviation when X is the number of heads that occur when you toss a coin five times.

Value of X (number of heads)	0	1	2	3	4	5
Probability	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$
$x_i p_i$						
$(x_i - \mu_X)^2 p_i$						

3. Repeat the same process to find the mean, variance and standard deviation when X is the number of heads that occur when you toss a coin six times.

Value of X (number of heads)	0	1	2	3	4	5	6
Probability	$\frac{1}{64}$	$\frac{3}{32}$	$\frac{15}{64}$	$\frac{10}{32}$	$\frac{15}{63}$	$\frac{3}{32}$	$\frac{1}{64}$
$x_i p_i$							
$(x_i - \mu_X)^2 p_i$							

4. Can you make a conjecture (guess) about the mean, variance and standard deviation for the number of heads in n tosses? *Hint: in each of the examples above, write the mean as a fraction over 2, the variance as a fraction over 4, and the standard deviation as a fraction over 2.*