

# Binomial Distributions

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We saw probability distributions for the random variable  $X$  that stood for the number of heads in 4, 5 or 6 coin tosses. In this example:

- 1 The outcome of each toss is either a “success” (heads) or a “failure” (tails).
- 2 There is a fixed number  $n$  of tosses.
- 3 Each toss is independent.
- 4 The probability of success is  $1/2$  for each toss.

# The Binomial Setting

Coin tossing is an example of the **binomial setting** - a situation in which the following four conditions are satisfied:

- 1 Each observation falls into one of two categories, one we call a “success” and the other we call a “failure.”
- 2 There is a fixed number  $n$  of observations.
- 3 The  $n$  observations are all **independent**, meaning that knowing the result of one observation does not tell you anything about the other observations.
- 4 The probability of success, which we write as  $p$ , is the same for each observation.

# Binomial Random Variables and Binomial Distributions

The random variable

$$X = \text{number of heads}$$

is an example of a binomial random variable, and its probability distribution is an example of a binomial distribution.

In a binomial setting, the random variable

$$X = \text{number of successes}$$

is called a **binomial random variable** and its probability distribution is called a **binomial distribution**.

# Recognizing the Binomial Setting Example 1: Blood Types

Blood type is inherited. If both parents carry genes for the O and A blood types, each child has probability 0.25 of getting two O genes and so having blood type O.

Suppose we study the number of O blood types among 5 children of these parents. Is this the binomial setting?

# Recognizing the Binomial Setting Example 2: Dealing Cards

Deal 10 cards from a shuffled deck and count the number  $X$  of red cards. Is this the binomial setting?

# Binomial Distributions in Statistical Sampling

Suppose an engineer chooses an SRS (simple random sample) of 10 switches from a shipment of 10,000 switches. Suppose (unknown to the engineer) that 10 % of the switches in the shipment are bad. The engineer counts  $X$  the number of bad switches.

This is not the binomial setting, since the observations are not independent. However, removing one switch from the shipment of 10,000 changes the makeup of the remaining 9999 very little.

Therefore, in practice the distribution of  $X$  is very close to the binomial distribution with  $n = 10$  and  $p = 0.1$ .

# Sampling Distribution of a Count

The bottom line from this example is:

Choose an SRS of size  $n$  from a population with proportion  $p$  of successes. When the population is **much larger** than the sample, the count  $X$  of successes in the sample has approximately the binomial distribution with parameters  $n$  and  $p$ .

# Binomial Coefficients

In order to find the probability distribution of a binomial random variable, we need binomial coefficients.

The number of ways of arranging  $k$  successes among  $n$  observations  $k = 0, 1, 2, \dots, n$  is given by the **binomial coefficient**:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

where  $n! = n(n-1)(n-2) \cdots (2)(1)$  and  $0! = 1$ .

The binomial coefficients are the numbers that show up in Pascal's triangle.

For example, how many ways is it possible to have 2 heads in 6 coin tosses?

# Binomial Probability

If  $X$  has the probability distribution with  $n$  observations and probability  $p$  of success on each observation, the possible values for  $X$  are  $0, 1, 2, \dots, n$  and for any such value  $k$ :

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}.$$

For example: compute the probability distribution where  $X$  is the number of heads in 4 coin tosses, 5 coin tosses, and 6 coin tosses.

# Example

Suppose you purchase a bundle of 10 bare-root broccoli plants. The sales clerk tells you that on average you can expect 5% of the plants to die before producing any broccoli. Assume that the bundle is a random sample of plants. Use the binomial formula to find the probability that you will lose at most 1 of the broccoli plants.

# Assignment

Assignment: Page 516, problems 8.3, 8.4, 8.5, and 8.6.

Page 519, problems 8.8 and 8.11.

Due Tuesday.