Algebra II, Block G
January 24, 2012
Proving Properties of Logarithms

Logarithms are defined as the inverse of an exponential:

Definition:
$$y = b^x \quad \text{if and only if} \quad \log_b y = x.$$

Using only the above definition, prove the following properties of logarithms:

1. $\log_b 1 = 0$

2. $\log_b b = 1$

3. $\log_b 0$ is undefined

4. $\log_b b^x = x$

Now we will prove some more interesting properties of logarithms, using the definition of logarithms, and properties we have already proven.

Property: $\log_b x^m = m \log_b x$.

Proof of Property:
Let $\log_b x = r$. Rewrite this expression using the definition of logarithms:

Therefore, $x^m =$ (simplify as much as possible)

And thus, $\log_b x^m =$ (simplify as much as possible)

Property: $\log_b(xy) = \log_b x + \log_b y$.

Proof of Property:
Let $\log_b x = r$. Rewrite this expression using the definition of logarithms:

Let $\log_b y = s$. Rewrite this expression using the definition of logarithms:

Thus $\log_b(xy) =$ (simplify as much as possible)

Property: $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$.

Proof of Property:

Property: $\log_b x = \frac{\log_a x}{\log_a b}$.

Proof of Property:

Property: $\log_a b = \frac{1}{\log_b a}$.

Proof of Property:

Property: $(\log_a b)(\log_b c) = \log_a c$.

Proof of Property: